Homework 11 - Math 142 **Name:** ____________________

1. The power series for $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$. Integrate this power series to find a power series for the antiderivative of $\ln(1+x)$.

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2. We also showed that $\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$. Use substitution to find a power series for the function $\arctan(2x^2)$.

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3. Let $f(x) = \ln\left(\frac{1+x}{1-x}\right)$. According to the logarithm rules: $\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$. Therefore, the derivative $f'(x) = \frac{1}{1+x} + \frac{1}{1-x}$. Find and simplify an infinite series for $f'(x)$ and then integrate to find an infinite series for $f(x)$.

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4. Re-write each series below using Σ-notation.

(a) $1 - \frac{1}{4} + \frac{1}{7} - \frac{1}{10} + \frac{1}{13} - \ldots.$

(b) $\frac{1}{10} + \frac{4}{100} + \frac{9}{1000} + \frac{16}{10000} + \ldots.$

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5. Show that each of the following series converges by finding a larger, simpler series that converges.

(a) $\sum_{n=2}^{\infty} \frac{n}{n^3 + 1}.$

(b) $\sum_{n=0}^{\infty} \frac{2^n - n^2}{3^n}.$

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6. Determine whether the infinite series $\sum_{n=0}^{\infty} \frac{1}{\ln(n+2)}$ converges or diverges by finding a good comparison. Explain how your comparison works.

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7. Explain clearly how you can tell that the following infinite series must diverge:

$$\frac{3}{5} + \frac{7}{6} + \frac{11}{7} + \frac{15}{8} + \frac{19}{9} + \ldots.$$
