

Calculus II - Math 142

Final Exam Review Problems

1. Evaluate the following integrals.

(a) $\int e^x \cos(e^x) dx.$

(b) $\int \tan^5 \theta \sec^3 \theta d\theta.$

(c) $\int x^2 \cos(3x) dx$

2. Find the third degree Taylor polynomial for $f(x) = x^3 + 2x - 3$ centered at $c = 2$.

3. Solve the differential equation $\frac{dy}{dx} = \frac{\cos x}{y^2}$ with initial condition $y(\pi) = 2$.

4. For each of the following series, determine whether it converges or diverges and give your reasoning.

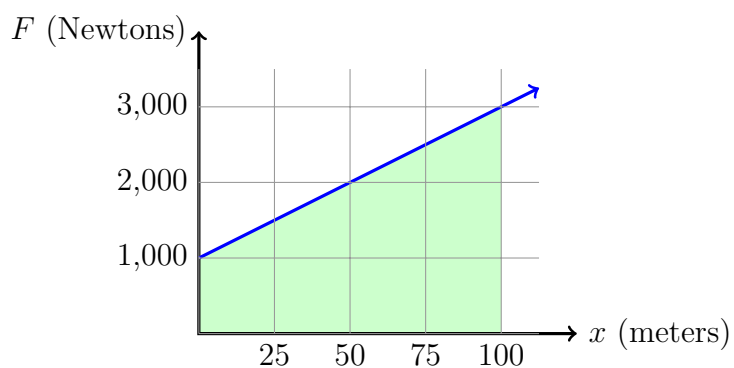
(a)
$$\sum_{n=0}^{\infty} \frac{(-1)^n 5^{n+1}}{6^n}$$

(b)
$$\sum_{k=2}^{\infty} \frac{\ln k}{k-1}$$

(c)
$$\sum_{n=1}^{\infty} \cos(n\pi)$$

5. Find all values of x for which the Taylor series $\sum_{n=0}^{\infty} \frac{2^n}{n} x^n$ converges.

6. Suppose I am pushing a heavy object over snow covered ground. The further I go, the deeper the snow gets, making me use more and more force to push the object. If the force I use as I push the object 100 meters is shown in the graph below, find the amount of work I did.

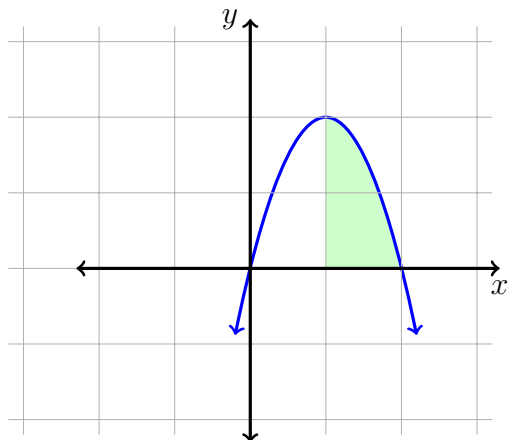


7. Find the following limits.

(a) $\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{x^2}$

(b) $\lim_{x \rightarrow \infty} \frac{e^x + \ln x}{x^2 + 100}$

8. Let \mathcal{R} be the region under the curve $y = 4x - 2x^2$ from $x = 1$ to $x = 2$.



- (a) Find the volume of the solid formed by revolving \mathcal{R} around the y -axis.
- (b) Set up, but do not evaluate, an integral for the volume of the solid formed by revolving \mathcal{R} around the x -axis.
9. Suppose that $f(x) = \sin(x^3)$.
- (a) Find a Maclaurin series for $f(x)$.
- (b) Use part (a) to find an infinite series for the integral $\int_0^1 \sin(x^3) dx$.

10. Evaluate the following integrals.

(a) $\int x^4 \ln x \, dx$

(b) $\int \frac{x^3 + 4}{x^2 - 4} \, dx$

11. Solve the following logarithm problems.

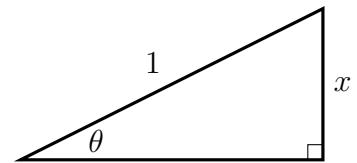
(a) Simplify $\log_5(50) + \log_5(\frac{5}{2})$.

(b) Solve the equation $2^{x-1} = e^5$.

12. Use logarithmic differentiation to find the derivative of $y = (1 + x)^x$.

13. Use the trig substitution $x = \sin \theta$ to evaluate

$$\int x^3 \sqrt{1 - x^2} \, dx$$



14. Simplify $\tan(\arcsin(x^2))$ using a reference triangle.

15. Find the area between the two curves $f(x) = x^2 - 6x$ and $g(x) = 3 - 4x$.

16. Estimate the worst case error in using the second degree Maclaurin polynomial $1 - \frac{x^2}{2}$ to approximate $\cos(0.3)$.

17. Find the sums of the following geometric series.

(a) $7 + 1 + \frac{1}{7} + \frac{1}{49} + \dots$

(b) $x^2 + \frac{x^3}{5} + \frac{x^4}{25} + \frac{x^5}{125} + \dots$

(c) $\sum_{n=0}^{\infty} \frac{(-3)^n}{4^{n-1}}$