Math 444 - Homework 12   **Name:** _____

1. Find the harmonic conjugate of $u(x, y) = x^2 - y^2$. That is, find a harmonic function $v(x, y)$ such that $u + iv$ is holomorphic.

2. Prove that if $u(x, y)$ is harmonic and bounded on $\mathbb{R}^2$, then $u(x, y)$ is constant. Hint: Recall that u must have a harmonic conjugate v. Use Liouville's theorem on $\exp(u + iv)$.

3. Give an example to show that the product of two harmonic functions is not necessarily harmonic.

4. Use power series to find the orders of the following zeros for the indicated functions.

   (a) $f(z) = 1 + \cos(z)$ at $z_0 = \pi$.

   (b) $g(z) = z^3 \sin(z^2)$ at $z_0 = 0$.