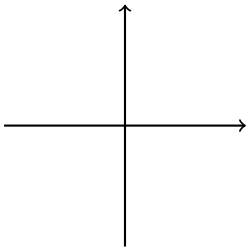
Math 444 - Homework 2 **Name:** _____

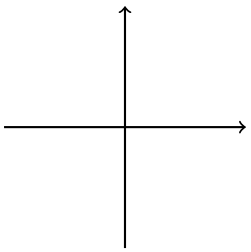
1. Describe the set $\{z \in \mathbb{C} : |z-1-i| = 2\}$ in words (using familiar concepts like circles, lines, distance, etc.), then use the axis provided to draw a picture of the set.



2. Circle the topological properties that apply to the set above, and cross out the ones that don't.

A. Open B. Closed C. Bounded D. Connected

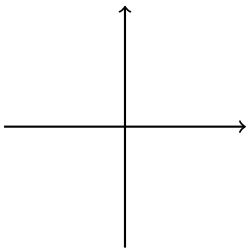
3. Describe the set $\{z \in \mathbb{C} : |z-2i| = |z|\}$ in words (using familiar concepts like circles, lines, distance, etc.), then use the axes provided to draw a picture of the set.



4. Circle the topological properties that apply to the set above, and cross out the ones that don't.

A. Open B. Closed C. Bounded D. Connected

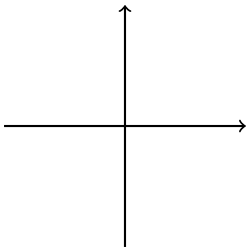
5. Describe the set $\{z \in \mathbb{C} : \mathrm{Im}\left(e^{i\pi/4}z\right) > 0\}$ in words (using familiar concepts like circles, lines, distance, etc.), then use the axes provided to draw a picture of the set.



6. Circle the topological properties that apply to the set above, and cross out the ones that don't.

A. Open B. Closed C. Bounded D. Connected

7. Describe the set $\{z \in \mathbb{C} : z^5 = 1\}$ in words (using familiar concepts like circles, lines, distance, etc.), then use the axes provided to draw a picture of the set.



8. Circle the topological properties that apply to the set above, and cross out the ones that don't.

A. Open B. Closed C. Bounded D. Connected

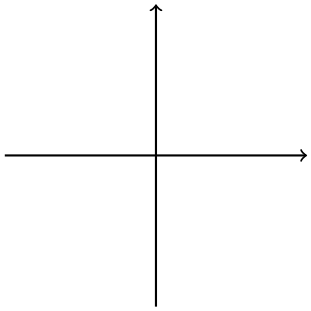
9. Give a parametrization of the line segment from i to 1.

10. Give a parametrization of the circle around $z = 2$ with radius 5, oriented clockwise.

11. Give a parametrization of the triangle with vertices $1 + i$, $i - 1$, and 0, oriented counter clockwise.

12. Let G be the union of the following two sets, $A = \{z \in \mathbb{C} : |z| < 2\}$ and $B = \{z \in \mathbb{C} : \mathrm{Im}(z) \leq 0\}$.

(a) Sketch G using the axes below.



(b) What is the boundary of G?

(c) What is the interior of G?

(d) Is G separated or connected? How can you tell?

13. Let z, w, a, b be complex numbers. Use the triangle inequality to prove that

$$|(z+w)-(a+b)| \leq |z-a| + |w-b|.$$

14. Use the triangle inequality to prove that if $|w-z| < \delta$, then $|w+z| < \delta + 2|z|$.

15. If $|w-z| < \delta$, show that $|w^2 - z^2| < \delta(\delta + 2|z|)$. Hint: The difference of squares formula works for complex numbers: $w^2 - z^2 = (w-z)(w+z)$.