**Math 444 - Homework 3** **Name:** ______________________

1. Show that $\text{Re}(z) = \dfrac{z + \overline{z}}{2}$ and $\text{Im}(z) = \dfrac{z - \overline{z}}{2i}$ for all $z \in \mathbb{C}$. Notice that the imaginary part of a complex number is actually a real number.

2. Let $a_n$, $b_n$ be sequences in $\mathbb{C}$. If $a_n$ converges to zero and $b_n$ is bounded, prove that the sequence $a_n \cdot b_n$ converges to zero.

3. Suppose $f$ and $g$ are complex-valued functions that are both continuous in a neighborhood of $a \in \mathbb{C}$. Prove that $f + g$ is continuous at $a$ by proving that for any sequence $z_n$ which converges to $a$, the sequence $f(z_n) + g(z_n)$ converges to $f(a) + g(a)$. Hint: You can reference the examples we did in the Sequences and Convergence workshop.

4. Sketch a graph of the two parametric curves $\gamma_1(s) = s + 4i$ and $\gamma_2(t) = 3 + it$ with $s, t \in \mathbb{R}$. Where do the curves intersect?

5. Let $f(z) = z^2$. Sketch a graph of the curve $f(2 + ti)$ for $t \in \mathbb{R}$.

6. Consider the image of the unit circle $\{z \in \mathbb{C} : |z| = 1\}$ under the map $g(z) = \dfrac{1}{z+2}$. Show that the resulting set is a circle and find its center and radius. **This problem is tough and my original hint wasn't great. Here is a better hint:** Find the center and radius by computing $g(-1)$ and $g(1)$. Then prove that $g(e^{i\theta})$ is always the same distance from the center.

7. Use the definition of derivative to show that the function $f(z) = \mathrm{Im}(z)$ is not differentiable anywhere.

8. Use the definition of derivative to find the derivative of $f(z) = \dfrac{1}{z}$.