Math 444 - Homework 6 **Name:** _____

Find all solutions to the following equations.

1. $\text{Log}(z) = i$

2. $\exp(z) = \pi i$

3. $z^5 = e^2$

4. $z^2 = 4 - 4i$

5. $z^n + 1 = 0$

6. Let $R = \{z \in \mathbb{C} : \frac{1}{e} < |z| < e\}$. Describe the shape of the image of R after you apply the principal logarithm to R? Draw a sketch of the set you get.

7. For the multiple-valued logarithm, there a difference between the set of all values of $\log(i^2)$ and the set of all values of $2\log i$. Find both sets and explain what it is.

8. Let $a, b \in \mathbb{C}$ with $a \neq 0$, and let $\log(\cdot)$ denote the multi-valued logarithm function. Prove that the set $\exp(b \log a)$ contains only a single value if and only if b is an integer. Hint: Find a formula for every element in the set $\exp(b \log a)$, and then prove that that formula gives only one value if and only if b is an integer.