Math 444 - Homework 7 **Name:** _____

1. In this problem you will evaluate the integral $\int_\gamma |z|^2\,dz$ where $\gamma(t)$ is the parabola $\gamma(t) = -t+i(t^2-1)$ from $t=-1$ to $t=1$.

   (a) What are the real and imaginary parts of $|\gamma(t)|^2 \cdot \gamma'(t)$?

   (b) Use the real and imaginary parts above to evaluate $\int_\gamma |z|^2\,dz$.

2. Integrate the function $z - \overline{z}$ on the upper half of the unit circle from $z=1$ to $z=-1$.

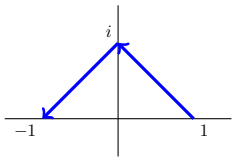
3. Find the length of the path $\gamma(t) = t + \frac{2}{3}t^{3/2}i$, $0 \le t \le 3$.

4. Show that $\lim_{n\to\infty} \left| \int_\gamma \frac{1}{z}\, dz \right| = 0$ when γ is the horizontal line segment from $1 - ni$ to $-1 - ni$. Hint: One way to do this would be to calculate the integral exactly for any n. An easier alternative is to use the inequality

$$\left| \int_\gamma f(z)\, dz \right| \leq \text{length}(\gamma) \cdot \max_{z \in \text{range}(\gamma)} |f(z)|.$$

Use a computer (I recommend Sympy) to calculate the following integrals.

5. $\int_\gamma (\overline{z})^3\, dz$ on the piecewise path shown below. Hint: in Sympy the complex conjugate function is `conjugate( )`. You'll need to parameterize each piece separately.



6. $\int_\gamma (\overline{z})^3\, dz$ on the piecewise path shown below.