Multiplication and Division

Lecture 17
Sections 3.1 - 3.4

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Outline

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1. Multiplication
   - Unsigned Multiplication
   - Signed Multiplication
2. Division
   - Unsigned Division
   - Signed Division
3. Assignment

The Multiplication Algorithm

8-bit Unsigned Multiplication

```
      01101001
    x 01101101
    ----------
      01101001
    01101001
   01101001
 01101001
01101001
--------------
10110010110101
```

- Notice that the product is nearly twice as long as the multiplicand and the multiplier.

The Multiplication Algorithm

- Because the product of two 32-bit integers can be as large as 64-bits, MIPS provides a register pair `hi` and `lo` to hold the product.
- MIPS provides two multiply instructions.
  - `mult` - signed multiply.
  - `multu` - unsigned multiply.
- They both put the 64-bit product into the registers `hi` and `lo`.
- The contents of `hi` and `lo` can be moved to general purpose registers using the instructions
  - `mfhi` - move from `hi`.
  - `mflo` - move from `lo`.

The Multiplication Algorithm

32-bit Unsigned Multiplication

`0x00000005` × `0x00000006`

`0x00000005` × `0xfffffffa`

`0xfffffffb` × `0xfffffffa`

- Run the program `unsigned_mult.asm` using the above examples and explain the results.

Outline

The Multiplication Algorithm

- Consider $x \times y$ using `mult` and `multu`.
- `mult` will multiply $x \times y$ and give the product the appropriate sign.
- `multu` will treat both integers as positive.

4-bit Signed Multiplication

4-bit Signed Multiplication

```
      1110  (-2)
    x 1101  (-3)
    ----
      1110
     0000
    1110
   1110
 -----
 10110110
```

The Multiplication Algorithm

- The two's complements of x and y are the integers $2^{32} - x$ and $2^{32} - y$.

$$\begin{aligned}(2^{32} - x)(2^{32} - y) &= 2^{64} - 2^{32}x - 2^{32}y + xy \\ &= 2^{32}(2^{32} - (x + y)) + xy.\end{aligned}$$

- Thus, `hi` will contain the two's complement of $x + y$ and `lo` will contain xy (assuming that $xy < 2^{32}$).
- What are the contents of `hi` and `lo` when x and $-y$ are multiplied by `multu`?

The Multiplication Algorithm

32-bit Signed Multiplication

`0x00000005` × `0x00000006`

`0x00000005` × `0xfffffffa`

`0xfffffffb` × `0xfffffffa`

- In the program `unsigned_mult.asm`, change `mulu` to `mul`.
- Run the program, using the above examples and explain the results.

Outline

Outline

The Division Algorithm

8-bit Unsigned Division

```
          10100 r 10
       --------
    101)1100110
        101
        ---
         010
         000
         ---
          101
          101
          ---
           001
           000
           ---
            010
            000
            ---
             10
```

- $102 \div 5 = 20$ rem 2.

MIPS Quotient and Remainder

- As we saw in the previous example, integer division results in two numbers: the quotient and the remainder.
- MIPS stores the quotient in `lo` and the remainder in `hi`.
- We then use `mfhi` or `mflo` to retrieve the one that we want.
- Note that the remainder of $a \div b$ is the same as $a \mod b$.

MIPS Division

- Run the program `int_div.asm`.

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1. Multiplication
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3. Assignment

MIPS Division

- Change `divu` to `div` in the program `int_div.asm` and run it again.
- Enter combinations of positive and negative numbers.

The Sign of the Quotient and Remainder

- When dividing a by b, the quotient q and remainder r must satisfy the following:
  - a, b, q, and r must satisfy the equation

$$a = qb + r.$$

  - The sign of q must be the same as the sign of ab.
  - The sign of r must be the same as the sign of a.
  - r must satisfy

$$0 \leq |r| < |b|.$$

- These four rules together determine the values of q and r.

The Sign of the Quotient and Remainder

Example (Positive $\div$ Positive)

- Let $a = +102$ and $b = +5$.
- Then q must be positive, r must be positive, and $0 \leq r < 5$.
- The only possible solution is

$$102 = (+20)(+5) + 2,$$

The Sign of the Quotient and Remainder

Example (Positive $\div$ Negative)

- Let $a = +102$ and $b = -5$.
- Then q must be negative, r must be positive, and $0 \le r < 5$.
- The only possible solution is

$$102 = (-20)(-5) + 2,$$

The Sign of the Quotient and Remainder

Example (Negative ÷ Positive)

- Let $a = -102$ and $b = +5$.
- Then q must be negative, r must be negative, and $0 \leq |r| < 5$.
- The only possible solution is

$$-102 = (-20)(+5) - 2,$$

The Sign of the Quotient and Remainder

Example (Negative ÷ Negative)

- Let $a = -102$ and $b = -5$.
- Then q must be positive, r must be negative, and $0 \leq |r| < 5$.
- The only possible solution is

$$-102 = (+20)(-5) - 2,$$

The Sign of the Quotient and Remainder

- This is why the mod operator `%` in C sometimes returns a negative value.
- In mathematics, on the other hand, the mod operator always produces a nonnegative value.

Outline

Assignment

Assignment

- Read Sections 3.1 - 3.4.