

# Turing Machine Examples

Lecture 27

Section 9.2

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- 1 Examples of Turing Machines
  - ADD – Add Two Integers
  - MULT – Multiply Two Integers
  - SQRT – Square Root

- 2 Assignment

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# ADD – Add Two Integers

## Example (ADD – Add Two Integers)

- Design a Turing machine ADD that will add two nonnegative integers, replacing them with their sum.

# ADD – Add Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .

# ADD – Add Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .

# ADD – Add Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- If  $n_2$  is not 0, then call DECR.



# ADD – Add Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- If  $n_2$  is not 0, then call DECR.
- Move left to  $n_1$ .

# ADD – Add Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- If  $n_2$  is not 0, then call DECR.
- Move left to  $n_1$ .
- Call INCR.

# ADD – Add Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- If  $n_2$  is not 0, then call DECR.
- Move left to  $n_1$ .
- Call INCR.
- Repeat until  $n_2$  is 0.

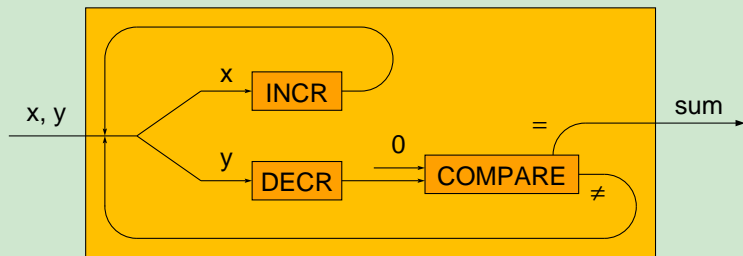
# ADD – Add Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- If  $n_2$  is not 0, then call DECR.
- Move left to  $n_1$ .
- Call INCR.
- Repeat until  $n_2$  is 0.
- Erase  $n_2$ .

# ADD – Add Two Integers

## Example (Strategy)



- 1 Examples of Turing Machines
  - ADD – Add Two Integers
  - **MULT – Multiply Two Integers**
  - SQRT – Square Root
  
- 2 Assignment

# MULT – Multiply Two Integers

## Example (MULT – Multiply Two Integers)

- Design a Turing machine MULT that will multiply two nonnegative integers and replace them with their product.

# MULT – Multiply Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .



# MULT – Multiply Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .

# MULT – Multiply Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- Call COPY. (Label the copy  $n_3$ .)

# MULT – Multiply Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- Call COPY. (Label the copy  $n_3$ .)
- Move right past  $n_3$ .

# MULT – Multiply Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- Call COPY. (Label the copy  $n_3$ .)
- Move right past  $n_3$ .
- Write 0. (Label it  $n_4$ .)

# MULT – Multiply Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- Call COPY. (Label the copy  $n_3$ .)
- Move right past  $n_3$ .
- Write 0. (Label it  $n_4$ .)
- Move left to  $n_3$ .

# MULT – Multiply Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- Call COPY. (Label the copy  $n_3$ .)
- Move right past  $n_3$ .
- Write 0. (Label it  $n_4$ .)
- Move left to  $n_3$ .
- Call ADD. (Compute  $n_3 + n_4$ ;  $n_4$  is gone.)

# MULT – Multiply Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- Call COPY. (Label the copy  $n_3$ .)
- Move right past  $n_3$ .
- Write 0. (Label it  $n_4$ .)
- Move left to  $n_3$ .
- Call ADD. (Compute  $n_3 + n_4$ ;  $n_4$  is gone.)
- Move left to  $n_1$ teger.

# MULT – Multiply Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- Call COPY. (Label the copy  $n_3$ .)
- Move right past  $n_3$ .
- Write 0. (Label it  $n_4$ .)
- Move left to  $n_3$ .
- Call ADD. (Compute  $n_3 + n_4$ ;  $n_4$  is gone.)
- Move left to  $n_1$ teger.
- Call INCR.



# MULT – Multiply Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- Call COPY. (Label the copy  $n_3$ .)
- Move right past  $n_3$ .
- Write 0. (Label it  $n_4$ .)
- Move left to  $n_3$ .
- Call ADD. (Compute  $n_3 + n_4$ ;  $n_4$  is gone.)
- Move left to  $n_1$ teger.
- Call INCR.
- Repeat until  $n_2$  is 0.

# MULT – Multiply Two Integers

## Example (Strategy)

- Label the integers  $n_1$  and  $n_2$ .
- Move right to  $n_2$ .
- Call COPY. (Label the copy  $n_3$ .)
- Move right past  $n_3$ .
- Write 0. (Label it  $n_4$ .)
- Move left to  $n_3$ .
- Call ADD. (Compute  $n_3 + n_4$ ;  $n_4$  is gone.)
- Move left to  $n_1$ teger.
- Call INCR.
- Repeat until  $n_2$  is 0.
- Erase  $n_2$ .

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape	
Input	$n_1$	$n_2$

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape		
Input	$n_1$	$n_2$	
Write 0	$n_1$	$n_2$	0

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape			
Input	$n_1$	$n_2$		
Write 0	$n_1$	$n_2$	0	
COPY	$n_1$	$n_2$	0	$n_2$

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape			
Input	$n_1$	$n_2$		
Write 0	$n_1$	$n_2$	0	
COPY	$n_1$	$n_2$	0	$n_2$
ADD	$n_1$	$n_2$	$n_2$	

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape			
Input	$n_1$	$n_2$		
Write 0	$n_1$	$n_2$	0	
COPY	$n_1$	$n_2$	0	$n_2$
ADD	$n_1$	$n_2$	$n_2$	
DECR	$n_1 - 1$	$n_2$	$n_2$	

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape			
Input	$n_1$	$n_2$		
Write 0	$n_1$	$n_2$	0	
COPY	$n_1$	$n_2$	0	$n_2$
ADD	$n_1$	$n_2$	$n_2$	
DECR	$n_1 - 1$	$n_2$	$n_2$	
COPY	$n_1 - 1$	$n_2$	$n_2$	$n_2$



# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape			
Input	$n_1$	$n_2$		
Write 0	$n_1$	$n_2$	0	
COPY	$n_1$	$n_2$	0	$n_2$
ADD	$n_1$	$n_2$	$n_2$	
DECR	$n_1 - 1$	$n_2$	$n_2$	
COPY	$n_1 - 1$	$n_2$	$n_2$	$n_2$
ADD	$n_1 - 1$	$n_2$	$2n_2$	

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape			
Input	$n_1$	$n_2$		
Write 0	$n_1$	$n_2$	0	
COPY	$n_1$	$n_2$	0	$n_2$
ADD	$n_1$	$n_2$	$n_2$	
DECR	$n_1 - 1$	$n_2$	$n_2$	
COPY	$n_1 - 1$	$n_2$	$n_2$	$n_2$
ADD	$n_1 - 1$	$n_2$	$2n_2$	
DECR	$n_1 - 2$	$n_2$	$2n_2$	

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape		
Input	$n_1$	$n_2$	
Write 0	$n_1$	$n_2$	0
COPY	$n_1$	$n_2$	0
ADD	$n_1$	$n_2$	$n_2$
DECR	$n_1 - 1$	$n_2$	$n_2$
COPY	$n_1 - 1$	$n_2$	$n_2$
ADD	$n_1 - 1$	$n_2$	$2n_2$
DECR	$n_1 - 2$	$n_2$	$2n_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
DECR	1	$n_2$	$(n_1 - 1)n_2$

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape			
Input	$n_1$	$n_2$		
Write 0	$n_1$	$n_2$	0	
COPY	$n_1$	$n_2$	0	$n_2$
ADD	$n_1$	$n_2$	$n_2$	
DECR	$n_1 - 1$	$n_2$	$n_2$	
COPY	$n_1 - 1$	$n_2$	$n_2$	$n_2$
ADD	$n_1 - 1$	$n_2$	$2n_2$	
DECR	$n_1 - 2$	$n_2$	$2n_2$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	
DECR	1	$n_2$	$(n_1 - 1)n_2$	
COPY	1	$n_2$	$(n_1 - 1)n_2$	$n_2$

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape			
Input	$n_1$	$n_2$		
Write 0	$n_1$	$n_2$	0	
COPY	$n_1$	$n_2$	0	$n_2$
ADD	$n_1$	$n_2$	$n_2$	
DECR	$n_1 - 1$	$n_2$	$n_2$	
COPY	$n_1 - 1$	$n_2$	$n_2$	$n_2$
ADD	$n_1 - 1$	$n_2$	$2n_2$	
DECR	$n_1 - 2$	$n_2$	$2n_2$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	
DECR	1	$n_2$	$(n_1 - 1)n_2$	
COPY	1	$n_2$	$(n_1 - 1)n_2$	$n_2$
ADD	1	$n_2$	$n_1 n_2$	

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape			
Input	$n_1$	$n_2$		
Write 0	$n_1$	$n_2$	0	
COPY	$n_1$	$n_2$	0	$n_2$
ADD	$n_1$	$n_2$	$n_2$	
DECR	$n_1 - 1$	$n_2$	$n_2$	
COPY	$n_1 - 1$	$n_2$	$n_2$	$n_2$
ADD	$n_1 - 1$	$n_2$	$2n_2$	
DECR	$n_1 - 2$	$n_2$	$2n_2$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	
DECR	1	$n_2$	$(n_1 - 1)n_2$	
COPY	1	$n_2$	$(n_1 - 1)n_2$	$n_2$
ADD	1	$n_2$	$n_1 n_2$	
DECR	0	$n_2$	$n_1 n_2$	

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape			
Input	$n_1$	$n_2$		
Write 0	$n_1$	$n_2$	0	
COPY	$n_1$	$n_2$	0	$n_2$
ADD	$n_1$	$n_2$	$n_2$	
DECR	$n_1 - 1$	$n_2$	$n_2$	
COPY	$n_1 - 1$	$n_2$	$n_2$	$n_2$
ADD	$n_1 - 1$	$n_2$	$2n_2$	
DECR	$n_1 - 2$	$n_2$	$2n_2$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	
DECR	1	$n_2$	$(n_1 - 1)n_2$	
COPY	1	$n_2$	$(n_1 - 1)n_2$	$n_2$
ADD	1	$n_2$	$n_1 n_2$	
DECR	0	$n_2$	$n_1 n_2$	
ERASE	$n_1 n_2$			

# MULT – Multiply Two Integers

## Example (MULT – Multiply Two Integers)

- Draw a diagram for MULT.



- 1 Examples of Turing Machines
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# SQRT – Square Root

## Example (SQRT – Square Root)

- Design a Turing machine SQRT that will find the square root of a nonnegative integer.
- By “square root” of  $n$ , we mean  $\lfloor \sqrt{n} \rfloor$ .
- For example,  $\text{SQRT}(20) = 4$ .

## Example (Strategy)

- Label the input  $n$ .
- Move right past  $n$ .
- Write 0. (Label it  $k$ .)
- Call COPY twice, making two copies of  $k$ .
- Call MULT, multiplying  $k \times k$ , producing  $k^2$ .
- Call COMPARE, comparing  $k^2$  to  $n$ .
- If  $k^2 \leq n$ , then INCR  $k$  and repeat previous three steps.
- If  $k^2 > n$ , then DECR  $k$  and quit.

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape
Input	$n$

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape
Input	$n$
Write 0	$n \quad 0$

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape
Input	$n$
Write 0	$n$ 0
COPY2	$n$ 0 0 0

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape
Input	$n$
Write 0	$n$ 0
COPY2	$n$ 0 0 0
MULT	$n$ 0 0

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape			
Input	$n$			
Write 0	$n$	0		
COPY2	$n$	0	0	0
MULT	$n$	0	0	
COMPARE	$n$	0	0	
ERASE	$n$	0		



# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape
Input	$n$
Write 0	$n$ 0
COPY2	$n$ 0 0 0
MULT	$n$ 0 0
COMPARE	$n$ 0 0
ERASE	$n$ 0
INCR	$n$ 1

INCR |  $n$  2

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape			
Input	$n$			
Write 0	$n$	0		
COPY2	$n$	0	0	0
MULT	$n$	0	0	
COMPARE	$n$	0	0	
ERASE	$n$	0		
INCR	$n$	1		
COPY2	$n$	1	1	1
INCR	$n$	2		
COPY2	$n$	2	2	2

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape			
Input	$n$			
Write 0	$n$	0		
COPY2	$n$	0	0	0
MULT	$n$	0	0	
COMPARE	$n$	0	0	
ERASE	$n$	0		
INCR	$n$	1		
COPY2	$n$	1	1	1
MULT	$n$	1	1	
COMPARE	$n$	1	1	
ERASE	$n$	1		
INCR	$n$	2		
COPY2	$n$	2	2	2
MULT	$n$	2	4	
COMPARE	$n$	2	4	
ERASE	$n$	2		

# MULT – Multiply Two Integers

## Example (Demo)

Action	Tape		
Input	$n$		
Write 0	$n$	0	
COPY2	$n$	0	0
MULT	$n$	0	0
COMPARE	$n$	0	0
ERASE	$n$	0	
INCR	$n$	1	
COPY2	$n$	1	1
MULT	$n$	1	1
COMPARE	$n$	1	1
ERASE	$n$	1	
INCR	$n$	2	
COPY2	$n$	2	2
MULT	$n$	2	4
COMPARE	$n$	2	4
ERASE	$n$	2	
⋮	⋮	⋮	⋮

# MULT – Multiply Two Integers

## Example (MULT – Multiply Two Integers)

- Draw a diagram for SQRT.

- 1 Examples of Turing Machines
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# Assignment

## Assignment

- The Turing machine DIV will receive a nonnegative integer  $x$  and positive integer  $y$  as input. Its output will be the greatest integer in  $x/y$ . Give a high-level description and a diagram for DIV.
- The Turing machine REM will receive a nonnegative integer  $x$  and positive integer  $y$  as input. Its output will be the remainder in  $x/y$ . Give a high-level description and a diagram for REM.