Math 111 **Homework #2** **Fall 2018**

1. Andy and Bob are playing the game of Chicken, with their cars. They drive their cars directly towards each other and their choices are to swerve or not to swerve. If at least one of them swerves, they will be no collision, but if neither one swerves, they will collide, destroying their vehicles and injuring themselves. If one swerves and the other does not, there is no collision, but the one who swerved has been humiliated. If they both swerve, then there is no cost to either one.

   (a) Is this a zero-sum game?

   (b) For each player, rank the four outcomes from best (1) to worst (4).

   (c) Assign your own dollar values (payoffs) to these outcomes and draw the game matrix. Because it is not a zero-sum game, for each outcome, you will need to assign payoffs to each player. Losses are negative payoffs.

   (d) Try to figure out the optimal strategy for Andy and Bob. Do they have optimal pure strategies?

2. Andy has two cards: a red 3 and a black 7. Bob also has two cards: a red 4 and a black 5. They each play one card.

   - If both cards are red, Andy wins the sum of the two numbers.
   - If neither card is red, Bob wins the sum of the two numbers.
   - If only one card is red, Andy wins the number on the red card.

   (a) Is this a zero-sum game?

   (b) Find the payoff to Andy for each outcome and draw the game matrix.

   (c) Do Andy and Bob have optimal pure strategies? If so, what are they? If not, explain why not.

3. Andy and Bob are playing Rock-Paper-Scissors. We all know how that works: Paper covers Rock; Scissors cut Paper; Rock smashes Scissors. Otherwise, it is a tie. Assume that when Andy wins, Bob pays him \$2, and when Bob wins, Andy pays him \$1. If they tie, then they pay nothing.

   (a) Is this a zero-sum game?

   (b) Draw the game matrix, showing the options (Rock, Paper, Scissors) and the payoffs.

   (c) Does either player have an optimal pure strategy?

4. The next day, Andy and Bob get together again to play Rock-Paper-Scissors, but this time, Andy has a really big rock, but also a really small piece of paper. Thus, Bob's normal paper cannot cover Andy's really big rock and Andy's really small paper cannot cover Bob's normal rock, so those combinations end in a tie. The other combinations are as before.

   (a) Draw the game matrix, showing the options and the payoffs.

   (b) Does either player have an optimal pure strategy? What happens if they both use their optimal strategies?

5. Andy and Bob are playing a game where Andy picks a number from 1 to 3 and Bob tries to guess the number. The payoffs are as follows.

   (i) If Bob guesses correctly, then Bob wins $10.

   (ii) If Bob guesses a larger number than Andy's number, then Andy wins the difference between the two numbers, in dollars.

   (iii) If Bob guesses a smaller number than Andy's number, then Bob wins the difference between the two numbers, in dollars.

   Then

   (a) Draw the game matrix, showing the options and the payoffs.

   (b) Does either player have an optimal pure strategy? What happens if they both use their optimal strategies?

6. In a game known as The Battle of the Sexes, Andy and Brandy have agreed to meet for a date either at the opera or at the football game. The trouble is, neither one can remember at which place they had agreed to meet. And they cannot communicate prior to arriving. (Their cell phones went dead.) So each one must decide where to go and hope that the other one is there. Now Andy would prefer the football game over the opera and Brandy would prefer the opera over the football game, but they would both prefer to be together rather than to end up at different places. If they do end up at different places, then Andy would rather be at the football game and Brandy would rather be at the opera than the other way around.

   (a) Is this a zero-sum game? If not, then you must list pairs of payoffs for each outcome.

   (b) Draw the game matrix, using your judgment to assign "dollar" values to the outcomes, based on their preferences. Use simple numbers.

   (c) Does either player have a pure optimal strategy?