# The Huntington-Hill Method

## Lecture 22
## Section 4.5

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1. The Huntington-Hill Method
   - Method 1
   - Method 2
2. Assignment

# Outline

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   - Method 1
   - Method 2
2. Assignment

# The Huntington-Hill Method

- In 1929, Congress set the size of the House of Representatives at 435 members.
- In 1941, Congress adopted the Huntington-Hill method for apportioning the seats in the House.
- Both laws remain in effect and will remain in effect for the foreseeable future.

# The Huntington-Hill Method

- There are two ways to apply the Huntington-Hill method.
- The first method, described in the textbook, involves guessing a modified divisor in a way similar to Jefferson's, Adams's, and Webster's methods.
- The second method, which is the one used by the government, involves no guesswork, but it may take longer to compute.

# Outline


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# The Huntington-Hill Method 1

- Compute the standard quotas $q_i$ for each state, as in the other methods.
- Round off the standard quota for each state by the following method.
  - Let $L$ be the lower quota and $U$ be the upper quota.
  - Compute the cutoff as $\sqrt{LU}$.
  - If $q_i < \sqrt{LU}$, then round down. Otherwise, round up.
  - The rounded value is the number of seats for that state.
- If the total number of seats is not $M$, then choose a modified divisor and repeat the procedure.

# Example

## Example (Example – Method 1)

- The populations of three states are $3, 7$ and 10 million people, respectively.
- The total number of seats apportioned to those states is 7.
- Use Method 1 to determine how many seats each state should get.

# Example

## Example (Example – Method 1)

- The total population is $P = 20$.
- The number of seats is $M = 7$.
- The standard divisor is $\text{SD} = \frac{20}{7} = 2.857$.

# Example

## Example (Example – Method 1)

| State | Pop ($p$) | $q = p/\text{SD}$ | $L$ | $U$ | $\sqrt{LU}$ | Seats |
|---|---|---|---|---|---|---|
| A | 3 | | | | | |
| B | 7 | | | | | |
| C | 10 | | | | | |

# Example

## Example (Example – Method 1)

| State | Pop ($p$) | $q = p/\text{SD}$ | $L$ | $U$ | $\sqrt{LU}$ | Seats |
|---|---|---|---|---|---|---|
| A | 3 | 1.05 | | | | |
| B | 7 | 2.45 | | | | |
| C | 10 | 3.50 | | | | |

# Example

## Example (Example – Method 1)

| State | Pop ($p$) | $q = p/\text{SD}$ | $L$ | $U$ | $\sqrt{LU}$ | Seats |
|---|---|---|---|---|---|---|
| A | 3 | 1.05 | 1 | 2 | $\sqrt{1 \cdot 2} = 1.414$ | |
| B | 7 | 2.45 | 2 | 3 | $\sqrt{2 \cdot 3} = 2.449$ | |
| C | 10 | 3.50 | 3 | 4 | $\sqrt{3 \cdot 4} = 3.464$ | |

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| A | 3 | 1.05 | 1 | 2 | $\sqrt{1 \cdot 2} = 1.414$ | 1 |
| B | 7 | 2.45 | 2 | 3 | $\sqrt{2 \cdot 3} = 2.449$ | 3 |
| C | 10 | 3.50 | 3 | 4 | $\sqrt{3 \cdot 4} = 3.464$ | 4 |

# Example

**Example (Example – Method 1)**

- The total number of seats apportioned is 8, so the “surplus” is $-1$.

# Example

**Example (Example – Method 1)**

- The total number of seats apportioned is 8, so the “surplus” is $-1$.
- We need a larger divisor.

# Example

**Example (Example – Method 1)**

- The total number of seats apportioned is 8, so the “surplus” is $-1$.
- We need a larger divisor.
- Let’s try $\text{MD} = 3.2$.

# Example

## Example (Example – Method 1)

| State | Pop ($p$) | $q = p/\text{MD}$ | $L$ | $U$ | $\sqrt{LU}$ | Seats |
|---|---|---|---|---|---|---|
| A | 3 | 0.937 | | | | |
| B | 7 | 2.187 | | | | |
| C | 10 | 3.125 | | | | |

## Example (Example – Method 1)

| State | Pop ($p$) | $q = p/\text{MD}$ | $L$ | $U$ | $\sqrt{LU}$ | Seats |
|---|---|---|---|---|---|---|
| A | 3 | 0.937 | 0 | 1 | $\sqrt{0 \cdot 1} = 0.000$ | |
| B | 7 | 2.187 | 2 | 3 | $\sqrt{2 \cdot 3} = 2.449$ | |
| C | 10 | 3.125 | 3 | 4 | $\sqrt{3 \cdot 4} = 3.464$ | |

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| B | 7 | 2.187 | 2 | 3 | $\sqrt{2 \cdot 3} = 2.449$ | 2 |
| C | 10 | 3.125 | 3 | 4 | $\sqrt{3 \cdot 4} = 3.464$ | 3 |

# Example

## Example (Example – Method 1)

- The total number of seats apportioned is 6, so the "surplus" is $+1$.

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**Example (Example – Method 1)**

- The total number of seats apportioned is 6, so the "surplus" is $+1$.
- Oops.

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- The total number of seats apportioned is 6, so the “surplus” is $+1$.
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- We need a smaller divisor.

# Example

## Example (Example – Method 1)

- The total number of seats apportioned is 6, so the "surplus" is $+1$.
- Oops.
- We need a smaller divisor.
- Let's try MD $= 2.86$.

# Example

## Example (Example – Method 1)

| State | Pop ($p$) | $q = p/\text{MD}$ | $L$ | $U$ | $\sqrt{LU}$ | Seats |
|---|---|---|---|---|---|---|
| A | 3 | 1.049 | | | | |
| B | 7 | 2.447 | | | | |
| C | 10 | 3.498 | | | | |

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| State | Pop ($p$) | $q = p/\text{MD}$ | $L$ | $U$ | $\sqrt{LU}$ | Seats |
|---|---|---|---|---|---|---|
| A | 3 | 1.049 | 1 | 2 | $\sqrt{1 \cdot 2} = 1.414$ | |
| B | 7 | 2.447 | 2 | 3 | $\sqrt{2 \cdot 3} = 2.449$ | |
| C | 10 | 3.498 | 3 | 4 | $\sqrt{3 \cdot 4} = 3.464$ | |

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# Outline


1. The Huntington-Hill Method
   - Method 1
   - Method 2

2. Assignment

# The Huntington-Hill Method

- Initially, every state gets a quota $q = 1$ (as required by the Constitution).
- Then divide each state's population $p$ by $D = \sqrt{q(q+1)}$, where $q$ is that state's current quota.
- The state with the largest such quotient gets one more seat, so add 1 to its quota $q$.
- Repeat the previous 2 steps until all the seats have been apportioned.

# Example

**Example (Example – Method 2)**

- The populations of three states are $3, 7$ and $10$ million people, respectively.
- The total number of seats apportioned to those states is 7.
- Use Method 2 to determine how many seats each state should get.

# Example

## Example (Example – Method 2)

| State | Population ($p$) | Seats ($q$) | $D = \sqrt{q(q+1)}$ | $p/D$ |
|---|---|---|---|---|
| A | 3 | 1 | $\sqrt{1 \cdot 2} = 1.414$ | $\frac{3}{\sqrt{2}} = 2.121$ |
| B | 7 | 1 | $\sqrt{1 \cdot 2} = 1.414$ | $\frac{7}{\sqrt{2}} = 4.949$ |
| C | 10 | 1 | $\sqrt{1 \cdot 2} = 1.414$ | $\frac{10}{\sqrt{2}} = 7.071$ |

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| B | 7 | 1 | $\sqrt{1 \cdot 2} = 1.414$ | $\frac{7}{\sqrt{2}} = 4.949$ |
| C | 10 | 2 | $\sqrt{1 \cdot 2} = 1.414$ | $\frac{10}{\sqrt{2}} = 7.071$ |

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| State | Population ($p$) | Seats ($q$) | $D = \sqrt{q(q+1)}$ | $p/D$ |
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| C | 10 | 2 | $\sqrt{2 \cdot 3} = 2.449$ | $\frac{10}{\sqrt{6}} = 4.082$ |

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| B | 7 | 2 | $\sqrt{2 \cdot 3} = 2.449$ | $\frac{7}{\sqrt{6}} = 2.857$ |
| C | 10 | 2 | $\sqrt{2 \cdot 3} = 2.449$ | $\frac{10}{\sqrt{6}} = 4.082$ |

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| B | 7 | 2 | $\sqrt{2 \cdot 3} = 2.449$ | $\frac{7}{\sqrt{6}} = 2.857$ |
| C | 10 | 3 | $\sqrt{3 \cdot 4} = 3.464$ | $\frac{10}{\sqrt{12}} = 2.886$ |

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# Which Method to Use?

- Suppose we had 3 states, with populations 2, 5, and 8 million, and 100 seats to apportion.
- Which method would be faster?

# Which Method to Use?

- Suppose we had 3 states, with populations 2, 5, and 8 million, and 100 seats to apportion.
- Which method would be faster?
- Why?

# Which Method to Use?

- Suppose we had 8 states, with populations 1, 2, 3, 4, 5, 6, 7, 8 million, and 9 seats to apportion.
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- Suppose we had 8 states, with populations 1, 2, 3, 4, 5, 6, 7, 8 million, and 9 seats to apportion.
- Which method would be faster?
- Why?

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1 The Huntington-Hill Method
- Method 1
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2 Assignment

# Assignment

## Assignment

- Ch. 4: Exercises 43, 45, 49. Use Method 1.
- Ch. 4: Exercises 49, 50. Use Method 2 with $M = 10$.