The Plurality and Borda Count Methods
Lecture 8
Sections 1.1 - 1.3

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Outline

Definitions

Definition (The Candidates)

The candidates are the people running for office in an election. If we are choosing something other than people, we call them alternatives.

Definition (The Voters)

The voters are the people who have a say in the outcome of the election. All votes count equally.

Definition (Single-choice Ballot)

In a single-choice ballot, each voter selects one candidate.

Definition (Preference Ballot)

In a preference ballot, each voter ranks all the candidates from most preferred to least preferred.

Definition (Truncated Preference Ballot)

In a truncated preference ballot, each voter ranks some, but not all, the candidates by preference.

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Definition (Truncated Preference Ballot)

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- We will use preference ballots (also called ranked choice ballots).

Outline

The Debate Club Election

Example

- There are four candidates for History Club president: A, B, C, and D.
- There are 19 voting members. Their preferences are shown on the next slide.

Example

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

The preferences.

Who won?

Example

- Who should be elected president?
- Who is more popular, A or B?
- Who is more popular, A or C?
- Who is more popular, A or D?
- Who is least popular?

Who won?

Example

- Who should be elected president?
- Who is more popular, A or B?
- Who is more popular, A or C?
- Who is more popular, A or D?
- Who is least popular?
- Do "least popular" and "most unpopular" mean the same thing?

Outline

The Plurality Method

Definition (The Plurality Method)

By the plurality method, the candidate with the most *first-place* votes wins.

Example

In the Debate Club example, A wins by the plurality method.

Web Page

Run the program Voting Methods on the web.

Outline

The Borda Count Method

Definition (The Borda Count Method)

By the Borda count method, the voters rank the candidates. Then each rank is assigned points, higher ranks receiving more points. The candidate with the *most points* wins.

The History Club Election

Example (The History Club Election)

- Reconsider the History Club election with 4 points for 1st, 3 for 2nd, 2 for 3rd, and 1 for 4th.

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

Example (The History Club Election)

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

The History Club Election

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

Points for $A : 9 \times 4 + 6 \times 1 + 4 \times 2 = 36 + 6 + 8 = 50$.

The History Club Election

Example (The History Club Election)

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

Points for A : $9 \times 4 + 6 \times 1 + 4 \times 2 = 36 + 6 + 8 = 50$.

Points for B : $9 \times 3 + 6 \times 3 + 4 \times 3 = 27 + 18 + 12 = 57$.

Example (The History Club Election)

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

Points for A : $9 \times 4 + 6 \times 1 + 4 \times 2 = 36 + 6 + 8 = 50$.

Points for B : $9 \times 3 + 6 \times 3 + 4 \times 3 = 27 + 18 + 12 = 57$.

Points for C : $9 \times 2 + 6 \times 4 + 4 \times 1 = 18 + 24 + 4 = 46$.

The History Club Election

Example (The History Club Election)

	9	6	4
1st	A	C	D
2nd	B	B	B
3rd	C	D	A
4th	D	A	C

Points for A : $9 \times 4 + 6 \times 1 + 4 \times 2 = 36 + 6 + 8 = 50$.

Points for B : $9 \times 3 + 6 \times 3 + 4 \times 3 = 27 + 18 + 12 = 57$.

Points for C : $9 \times 2 + 6 \times 4 + 4 \times 1 = 18 + 24 + 4 = 46$.

Points for D : $9 \times 1 + 6 \times 2 + 4 \times 4 = 9 + 12 + 16 = 37$.

Example (The History Club Election)

- Which candidate wins?

The History Club Election

Example (The History Club Election)

- Which candidate wins?
- Which candidate comes in last?

The History Club Election

> **Example (The History Club Election)**
>
> - Which candidate wins?
> - Which candidate comes in last?
> - Would the outcome be different if the points were 3, 2, 1, 0?

The History Club Election

Example (The History Club Election)

- Which candidate wins?
- Which candidate comes in last?
- Would the outcome be different if the points were 3, 2, 1, 0?
- What about 20, 15, 10, 5?

The History Club Election

Example (The History Club Election)

- Which candidate wins?
- Which candidate comes in last?
- Would the outcome be different if the points were 3, 2, 1, 0?
- What about 20, 15, 10, 5?
- What about 5, 4, 3, 0?

Outline

Burying a Candidate

Burying a Candidate

- The Borda-count method is susceptible to chicanery.
- If the voters vote "honestly," then there is no problem.
- But what if. . . ?

Burying a Candidate

Example (Burying a Candidate)

- There are three candidates: Democrat (D), Republican (R), and Wacko (W).
- There are 9 Democratic voters, 7 Republican voters, and 4 Wacko voters.
- Their preferences:

	9	7	2	2
1st	D	R	W	W
2nd	R	D	R	D
3rd	W	W	D	R

- Who wins?

Burying a Candidate

Example (Burying a Candidate)

- There are three candidates: Democrat (D), Republican (R), and Wacko (W).
- There are 9 Democratic voters, 7 Republican voters, and 4 Wacko voters.
- Their preferences:

	9	7	2	2
1st	D	R	W	W
2nd	R	D	R	D
3rd	W	W	D	R

- Who wins?
- D wins.

Burying a Candidate

Example (Burying a Candidate)

- What if the Republicans decide to "bury" the Democrat?
- Their preferences:

	9	7	2	2
1st	D	R	W	W
2nd	R	D	R	D
3rd	W	W	D	R

- . . .

Burying a Candidate

Example (Burying a Candidate)

- What if the Republicans decide to "bury" the Democrat?
- Their false preferences:

	9	7	2	2
1st	D	R	W	W
2nd	R	W	R	D
3rd	W	D	D	R

- Now who wins?

Burying a Candidate

Example (Burying a Candidate)

- What if the Republicans decide to "bury" the Democrat?
- Their false preferences:

	9	7	2	2
1st	D	R	W	W
2nd	R	W	R	D
3rd	W	D	D	R

- Now who wins? R wins because D is "buried."

Burying a Candidate

Example (Burying a Candidate)

- What if, in addition, the Democrats decide to "bury" the Republican?
- Their preferences:

	9	7	2	2
1st	D	R	W	W
2nd	R	W	R	D
3rd	W	D	D	R

Burying a Candidate

Example (Burying a Candidate)

- What if, in addition, the Democrats decide to "bury" the Republican?
- Their false preferences:

	9	7	2	2
1st	D	R	W	W
2nd	W	W	R	D
3rd	R	D	D	R

Example (Burying a Candidate)

- What if, in addition, the Democrats decide to "bury" the Republican?
- Their false preferences:

	9	7	2	2
1st	D	R	W	W
2nd	W	W	R	D
3rd	R	D	D	R

- Now who wins? (D and R are both "buried.")

Burying a Candidate

Example (Burying a Candidate)

- What if, in addition, the Democrats decide to "bury" the Republican?
- Their false preferences:

	9	7	2	2
1st	D	R	W	W
2nd	W	W	R	D
3rd	R	D	D	R

- Now who wins? (D and R are both "buried.") Wacko wins! Oops!

Outline

Assignment

Assignment

- Chapter 1: Exercises 11, 13, 15, 16, 21, 25, 27, 29 (8th ed.).
- For 9th ed., see the worksheet online.