

The Pairwise-Comparison Method

Lecture 10
Section 1.5

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- 1 The Method of Pairwise Comparisons
- 2 Examples
- 3 The Number of Comparisons
- 4 A Shortcoming of the Method
- 5 Assignment

Outline

- 1 The Method of Pairwise Comparisons
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The Method of Pairwise Comparisons

Definition (The Method of Pairwise Comparisons)

By the **method of pairwise comparisons**, each voter ranks the candidates. Then, **for every pair** (for every possible two-way race) of candidates,

- Determine which one was preferred more often.
- That candidate gets 1 point.
- If there is a tie, each candidate gets $1/2$ point.

The candidate who gets the greatest number of points is the winner.

Then rank the candidates according to the number of points received.

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- 1 The Method of Pairwise Comparisons
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Example

Example

Suppose that there are 3 candidates: A, B, C. The following table summarizes the voters' preferences.

No. of voters	Preferences			
	7	6	3	2
1st	A	B	B	C
2nd	B	A	C	B
3rd	C	C	A	A

- How many pairings are there?
- List the pairings.
- Count the votes for each pairing and determine the winner.

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Listing Pairings

- Candidates A, B, C, D, . . .
- List all pairings with A, starting with AB.
- Then list all pairings with B, starting with BC.
- Then all pairings with C, starting with CD, and so on.

Listing Pairings

Pairings of A, B, C

Pairings of A, B, C.

Listing Pairings

Pairings of A, B, C

AB

AC

Pairings of A, B, C.

Listing Pairings

Pairings of A, B, C

AB BC
AC

Pairings of A, B, C.

Listing Pairings

Pairings of A, B, C, D

Pairings of A, B, C, D.

Listing Pairings

Pairings of A, B, C, D

AB

AC

AD

Pairings of A, B, C, D.

Listing Pairings

Pairings of A, B, C, D

AB BC
AC BD
AD

Pairings of A, B, C, D.

Listing Pairings

Pairings of A, B, C, D

AB BC CD
AC BD
AD

Pairings of A, B, C, D.

Listing Pairings

Pairings of A, B, C, D, E

Pairings of A, B, C, D, E.

Listing Pairings

Pairings of A, B, C, D, E

AB

AC

AD

AE

Pairings of A, B, C, D, E.

Listing Pairings

Pairings of A, B, C, D, E

AB BC
AC BD
AD BE
AE

Pairings of A, B, C, D, E.

Listing Pairings

Pairings of A, B, C, D, E

AB	BC	CD
AC	BD	CE
AD	BE	
AE		

Pairings of A, B, C, D, E.

Listing Pairings

Pairings of A, B, C, D, E

AB	BC	CD	DE
AC	BD	CE	
AD	BE		
AE			

Pairings of A, B, C, D, E.

Example

Example

Suppose that there are 4 candidates: A, B, C, D. The following table summarizes the voters' preferences.

No. of voters	Preferences			
	11	8	7	4
1st	A	B	D	C
2nd	B	D	A	A
3rd	C	C	B	D
4th	D	A	C	B

- How many pairings are there?
- List the pairings.
- Count the votes for each pairing and determine the winner.

The Number of Comparisons

- How many comparisons are there?
 - With 6 candidates, how many comparisons are there?

The Number of Comparisons

- How many comparisons are there?
 - With 6 candidates, how many comparisons are there? 15

The Number of Comparisons

- How many comparisons are there?
 - With 6 candidates, how many comparisons are there? 15
 - With 7 candidates, how many comparisons are there?

The Number of Comparisons

- How many comparisons are there?
 - With 6 candidates, how many comparisons are there? 15
 - With 7 candidates, how many comparisons are there? 21

The Number of Comparisons

- How many comparisons are there?
 - With 6 candidates, how many comparisons are there? 15
 - With 7 candidates, how many comparisons are there? 21
 - With 10 candidates, how many comparisons are there?

The Number of Comparisons

- How many comparisons are there?
 - With 6 candidates, how many comparisons are there? 15
 - With 7 candidates, how many comparisons are there? 21
 - With 10 candidates, how many comparisons are there? 45

The Number of Comparisons

- How many comparisons are there?
 - With 6 candidates, how many comparisons are there? 15
 - With 7 candidates, how many comparisons are there? 21
 - With 10 candidates, how many comparisons are there? 45
 - In general, with N candidates, there are $\frac{1}{2}N(N - 1)$ comparisons.

Example

Example

Suppose that there are 5 candidates: A, B, C, D, E. The following table summarizes the voters' preferences.

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	E	E
2nd	A	A	A	E	D	B	D
3rd	E	D	C	D	E	A	A
4th	D	C	E	B	B	D	B
5th	C	E	B	A	C	C	C

- How many pairings are there?

Example

Example

Suppose that there are 5 candidates: A, B, C, D, E. The following table summarizes the voters' preferences.

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	E	E
2nd	A	A	A	E	D	B	D
3rd	E	D	C	D	E	A	A
4th	D	C	E	B	B	D	B
5th	C	E	B	A	C	C	C

- How many pairings are there? 10

Example

Example

Suppose that there are 5 candidates: A, B, C, D, E. The following table summarizes the voters' preferences.

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	E	E
2nd	A	A	A	E	D	B	D
3rd	E	D	C	D	E	A	A
4th	D	C	E	B	B	D	B
5th	C	E	B	A	C	C	C

- How many pairings are there? 10
- List the pairings.

Example

Example

Suppose that there are 5 candidates: A, B, C, D, E. The following table summarizes the voters' preferences.

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	E	E
2nd	A	A	A	E	D	B	D
3rd	E	D	C	D	E	A	A
4th	D	C	E	B	B	D	B
5th	C	E	B	A	C	C	C

- How many pairings are there? **10**
- List the pairings. **AB, AC, AD, AE, BC, BD, BE, CD, CE, DC**

Example

Example

Suppose that there are 5 candidates: A, B, C, D, E. The following table summarizes the voters' preferences.

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	E	E
2nd	A	A	A	E	D	B	D
3rd	E	D	C	D	E	A	A
4th	D	C	E	B	B	D	B
5th	C	E	B	A	C	C	C

- How many pairings are there? **10**
- List the pairings. **AB, AC, AD, AE, BC, BD, BE, CD, CE, DC**
- Who is the winner?

Example

Example

Suppose that there are 5 candidates: A, B, C, D, E. The following table summarizes the voters' preferences.

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	E	E
2nd	A	A	A	E	D	B	D
3rd	E	D	C	D	E	A	A
4th	D	C	E	B	B	D	B
5th	C	E	B	A	C	C	C

- How many pairings are there? **10**
- List the pairings. **AB, AC, AD, AE, BC, BD, BE, CD, CE, DC**
- Who is the winner? **A**

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A Shortcoming

- This method seems to take pretty much everything into account.
- So what could go wrong?

A Shortcoming

Example (A Shortcoming)

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	E	E
2nd	A	A	A	E	D	B	D
3rd	E	D	C	D	E	A	A
4th	D	C	E	B	B	D	B
5th	C	E	B	A	C	C	C

- Reconsider the previous example.

A Shortcoming

Example (A Shortcoming)

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	E	E
2nd	A	A	A	E	D	B	D
3rd	E	D	C	D	E	A	A
4th	D	C	E	B	B	D	B
5th	C	E	B	A	C	C	C

- At the last minute, candidate E drops out.

A Shortcoming

Example (A Shortcoming)

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	B	D
2nd	A	A	A	D	D	A	A
3rd	D	D	C	B	B	D	B
4th	C	C	B	A	C	C	C

- Now who is the winner?

A Shortcoming

Example (A Shortcoming)

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	B	D
2nd	A	A	A	D	D	A	A
3rd	D	D	C	B	B	D	B
4th	C	C	B	A	C	C	C

- Now who is the winner? **B**

A Shortcoming

Example (A Shortcoming)

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	B	D
2nd	A	A	A	D	D	A	A
3rd	D	D	C	B	B	D	B
4th	C	C	B	A	C	C	C

- Now who is the winner? **B**
- Is that surprising?

A Shortcoming

Example (A Shortcoming)

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	C	A	B	D
2nd	A	A	A	D	D	A	A
3rd	D	D	C	B	B	D	B
4th	C	C	B	A	C	C	C

- Now who is the winner? **B**
- Is that surprising?
- This is considered to be a shortcoming of the method. Do you agree?

The Perfect Voting Method

- Is there a voting method that has no shortcoming?

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Assignment

Assignment

- Chapter 1: Exercises 41, 42, 43, 44, 45, 47, 49, 50.