Weighted Voting

Lecture 12
Section 2.1

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Outline

Introduction

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- Would it ever be fair to give one voter more votes than another voter?

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- Yes.

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- How much "influence" does each partner have?
- What if decisions are made by a simple majority (11 votes)?

Outline

Definitions

Definition (The Players)

The players are the same as the voters. Let N denote the number of players.

Definition (The Weights)

The weight of a player is the number of votes that he may cast. The weights are denoted $w_1, w_2, w_3, \ldots, w_N$. The total of the weights is $V = w_1 + w_2 + w_3 + \cdots + w_N$.

Definition (The Quota)

The quota q is the number of votes needed to win.

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The quota, denoted q, is the number of votes needed to pass a motion.

- We represent the voting system as $[q : w_1, w_2, \ldots, w_N]$.
- The previous examples the voting systems were $[14 : 9, 8, 3, 1]$ and $[11 : 9, 8, 3, 1]$.

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- Thus, we want $q > V/2$.
- Might there be a good reason to set $q \leq V/2$?

Gridlock

Example (Gridlock)

Change the quota to 22: $[22 : 9, 8, 3, 1]$. Now we have “gridlock.”

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Definition (Gridlock)

Gridlock occurs when $q > V$.

Thus, we always want $V/2 < q \leq V$.

Dictators

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If Andy buys 5 shares from Bob, then the situation becomes $[14 : 14, 3, 3, 1]$ and Andy becomes a “dictator.”

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Definition (Dictator)

A dictator is a player whose weight is greater than or equal to q. He can pass a motion by himself.

Avoid Dictators

- To avoid dictators, we need $w_i < q$ for every i.
  - No single voter's weight is enough to pass a motion.
  - Equivalently, $q > w_i$ for every i.

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  - Equivalently, $q > w_i$ for every i.
  - Equivalently, q is greater than the *greatest* weight.

Veto Power

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Definition (Veto Power)

A player has veto power if the sum of *all other* votes is less than q. That is $V - w_i < q$. In such a case, no motion can pass unless that player votes for it.

Avoid Veto Power

- To avoid veto power, we need $V - w_i \geq q$ for every i.
  - That is, no single voter's weight is so much that no coalition can pass a motion without his vote.
  - Equivalently, $q \leq V - w_i$ for every i.

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  - Equivalently, $q \leq V - w_i$ for every i.
  - Equivalently, q is less than or equal to $V-$ the *largest* weight.

Dictators and Veto Power

Example

- In the voting system $[q : 10, 7, 6, 5, 3]$,
  - What values of q will avoid anarchy?
  - What values of q will avoid gridlock?
  - What values of q will prevent dictators?
  - What values of q will avoid veto power?

Dictators and Veto Power

Example

- In the voting system $[q : 10, 7, 6, 5, 3]$,
  - What values of q will avoid anarchy? $q \geq 16$
  - What values of q will avoid gridlock?
  - What values of q will prevent dictators?
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  - What values of q will avoid veto power? $q \leq 21$

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  - What values of q will avoid veto power? $q \leq 21$
- Thus, we want $16 \leq q \leq 21$.

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Example (Few Votes, Much Power)

- Consider the situation $[19 : 8, 7, 3, 2]$.
- What combinations of players will achieve 19 votes?

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- All players are equally powerful.

Example (Few Votes, Much Power)

- Consider the situation $[19 : 8, 7, 3, 2]$.
- What combinations of players will achieve 19 votes? ABC, ABD, ACD, BCD, ABCD
- All players are equally powerful.
- The voting system might as well be $[4 : 1, 1, 1, 1]$.

Example (Many Votes, Little Power)

- Consider the situation $[18 : 6, 6, 6, 5]$.
- What combinations of players will achieve 18 votes?

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- D's vote does not make any difference.

Example (Many Votes, Little Power)

- Consider the situation $[18 : 6, 6, 6, 5]$.
- What combinations of players will achieve 18 votes? ABC, ABCD
- D's vote does not make any difference.
- The voting system might as well be $[3 : 1, 1, 1, 0]$.

Outline

Assignment

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- Ch. 2: Exercises 1, 2, 3, 4, 5, 6, 7, 8.