

# Shapley-Shubik Power

Lecture 14  
Section 2.3

Robb T. Koether

Hampden-Sydney College

Fri, Sep 28, 2018

- 1 Introduction
- 2 Definitions
- 3 Listing Permutations
- 4 Pairs vs. Coalitions vs. Sequential Coalitions
- 5 Shapley-Shubik Power
- 6 Examples
- 7 The Electoral College
- 8 Assignment

# Outline

- 1 Introduction
- 2 Definitions
- 3 Listing Permutations
- 4 Pairs vs. Coalitions vs. Sequential Coalitions
- 5 Shapley-Shubik Power
- 6 Examples
- 7 The Electoral College
- 8 Assignment

# Introduction

In the national political conventions, when the role is called for votes, the state delegations vie for the honor of being the state that puts their candidate “over the top.”

- Does it really matter?

# Introduction

In the national political conventions, when the role is called for votes, the state delegations vie for the honor of being the state that puts their candidate “over the top.”

- Does it really matter?
- No.

# Outline

- 1 Introduction
- 2 Definitions**
- 3 Listing Permutations
- 4 Pairs vs. Coalitions vs. Sequential Coalitions
- 5 Shapley-Shubik Power
- 6 Examples
- 7 The Electoral College
- 8 Assignment

# Definitions

## Definition (Sequential Coalition)

A **sequential coalition** is an ordered list of all the players.

If there are  $N$  players, then there are  $N!$  sequential coalitions.

## Definition (Pivotal Player)

When counting up the votes, from left to right, in a sequential coalition, the player who *first* makes the total reach the quota is the **pivotal player** for that sequence.

# Definitions

## Definition (Sequential Coalition)

A **sequential coalition** is an ordered list of all the players.

If there are  $N$  players, then there are  $N!$  sequential coalitions.

## Definition (Pivotal Player)

When counting up the votes, from left to right, in a sequential coalition, the player who *first* makes the total reach the quota is the **pivotal player** for that sequence.

Every sequential coalition will have exactly one pivotal player.



# Outline

- 1 Introduction
- 2 Definitions
- 3 Listing Permutations**
- 4 Pairs vs. Coalitions vs. Sequential Coalitions
- 5 Shapley-Shubik Power
- 6 Examples
- 7 The Electoral College
- 8 Assignment

# Listing Permutations

- Given two players, there two permutations:

AB BA

# Listing Permutations

- Given two players, there are two permutations:

AB BA

- Given three players, there are six permutations:

ABC BAC CAB  
ACB BCA CBA

# Listing Permutations

- Given two players, there are two permutations:

AB BA

- Given three players, there are six permutations:

ABC BAC CAB  
ACB BCA CBA

# Listing Permutations

- Given two players, there are two permutations:

AB BA

- Given three players, there are six permutations:

ABC BAC CAB  
ACB BCA CBA

# Listing Permutations

- Given two players, there are two permutations:

AB BA

- Given three players, there are six permutations:

ABC BAC CAB  
ACB BCA CBA

- What about four players?

# Listing Permutations

ABCD	BACD	CABD	DABC
ABDC	BADC	CADB	DACB
ACBD	BCAD	CBAD	DBAC
ACDB	BCDA	CBDA	DBCA
ADBC	BDAC	CDAB	DCAB
ADCB	BDCA	CDBA	DCBA

Four players

# Listing Permutations

ABCD	BACD	CABD	DABC
ABDC	BADC	CADB	DACB
ACBD	BCAD	CBAD	DBAC
ACDB	BCDA	CBDA	DBC A
ADBC	BDAC	CDAB	DCAB
ADCB	BDCA	CDBA	DCBA

Four players



# Listing Permutations

ABCD	BACD	CABD	DABC
ABDC	BADC	CADB	DACB
ACBD	BCAD	CBAD	DBAC
ACDB	BCDA	CBDA	DBCA
ADBC	BDAC	CDAB	DCAB
ADCB	BDCA	CDBA	DCBA

Four players

# Listing Permutations

ABCDE	ADBCE	BACDE	BDACE	CABDE	CDABE	DABCE	DCABE	EABCD	ECABD
ABCED	ADBEC	BACED	BDAEC	CABED	CDAEB	DABEC	DCAEB	EABDC	ECABD
ABDCE	ADCBE	BADCE	BDCAE	CADBE	CDBAE	DACBE	DCBAE	EACBD	ECBAD
ABDEC	ADCEB	BADEC	BDCEA	CADEB	CDBEA	DACEB	DCBEA	EACDB	ECBDA
ABECD	ADEBC	BAECD	BDEAC	CAEBD	CDEAB	DAEBC	DCEAB	EADBC	ECBAD
ABEDC	ADECB	BAEDC	BDECA	CAEDB	CDEBA	DAECB	DCEBA	EADCB	ECBDA
ACBDE	AEBCD	BCADE	BEACD	CBADE	CEABD	DBACE	DEABC	EBACD	EDABC
ACBED	AEBDC	BCAED	BEADC	CBAED	CEADB	DBAEC	DEACB	EBADC	EDACB
ACDBE	AECBD	BCDAE	BECAD	CBDAE	CEBAD	DBCAE	DEBAC	EBCAD	EDBAC
ACDEB	AECDB	BCDEA	BECDA	CBDEA	CEBDA	DBCEA	DEBCA	EBCDA	EDBCA
ACEBD	AEDBC	BCEAD	BEDAC	CBEAD	CEADB	DBEAC	DECAB	EBDAC	EDCAB
ACEDB	AEDCB	BCEDA	BEDCA	CBEDA	CEDBA	DBECA	DECBA	EBDCA	EDCBA

Five players

# Listing Permutations

ABCDE	ADBCE	BACDE	BDACE	CABDE	CDABE	DABCE	DCABE	EABCD	ECABD
ABCED	ADBEC	BACED	BDAEC	CABED	CDAEB	DABEC	DCAEB	EABDC	ECABD
ABDCE	ADCBE	BADCE	BDCAE	CADBE	CDBAE	DACBE	DCBAE	EACBD	ECBAD
ABDEC	ADCEB	BADEC	BDCEA	CADEB	CDBEA	DACEB	DCBEA	EACDB	ECBDA
ABECD	ADEBC	BAECD	BDEAC	CAEBD	CDEAB	DAEBC	DCEAB	EADBC	ECBAD
ABEDC	ADECB	BAEDC	BDECA	CAEDB	CDEBA	DAECB	DCEBA	EADCB	ECBDA
ACBDE	AEBCD	BCADE	BEACD	CBADE	CEABD	DBACE	DEABC	EBACD	EDABC
ACBED	AEBDC	BCAED	BEADC	CBAED	CEADB	DBAEC	DEACB	EBADC	EDACB
ACDBE	AECBD	BCDAE	BECAD	CBDAE	CEBAD	DBCAE	DEBAC	EBCAD	EDBAC
ACDEB	AECDB	BCDEA	BECDA	CBDEA	CEBDA	DBCEA	DEBCA	EBCDA	EDBCA
ACEBD	AEDBC	BCEAD	BEDAC	CBEAD	CEADB	DBEAC	DECAB	EBDAC	EDCAB
ACEDB	AEDCB	BCEDA	BEDCA	CBEDA	CEDBA	DBECA	DECBA	EBDCA	EDCBA

Five players

# Six Players

ABCDEF	ACBDEF	ADBCEF	AEBCDF	AFBCDE	BACDEF	BCADEF	BDACEF	BEACDF	BFACDE
ABCD FE	ACBD FE	ADBC FE	AEB CDF	AFBC ED	BAC DFE	BCAD FE	BDAC FE	BEAC FD	BFAC ED
ABCEDF	ACBEDF	ADBECF	AEBDCF	AFBDCE	BACEDF	BCAEDF	BDAECF	BEADCF	BFADCE
ABCEFD	ACBEFD	ADBEFC	AEBDFC	AFBDEC	BACEFD	BCAEFD	BDAEFC	BEADFC	BFAD EC
ABC FDE	ACB FDE	ADB FCE	AEB FCD	AFB ECD	BAC FDE	BCAF DE	BDA FCE	BEAF CD	BFAC ED
ABC FED	ACB FED	ADB FEC	AEB FDC	AFB EDC	BAC FED	BCAF ED	BDA FEC	BEAF DC	BFAC ED
ABDCEF	ACDBEF	ADCBEF	AECBDF	AFCBDE	BADCEF	BCDAEF	BDCAEF	BECADF	BFCADE
ABDCFE	ACDBFE	ADCBEF	AECBDF	AFCBED	BADCFE	BCDAFE	BDCAFE	BECAFD	BFCAED
ABDECF	ACDEBF	ADCEBF	AECDBF	AFCD BE	BADECF	BCDEAF	BDCEAF	BECDAF	BFCDAE
ABDEF C	ACDEF B	ADCEFB	AECDFB	AFCD EB	BADEF C	BCDEF A	BDCEFA	BECDFA	BFCDEA
ABDFCE	ACDFBE	ADCFBE	AECFBD	AFCEBD	BADFCE	BCDFAE	BDCFAE	BECFAD	BFCEAD
ABDFEC	ACDFEB	ADCFEB	AECFDB	AFCEDB	BADFEC	BCDFEA	BDCFEA	BECFDA	BFCEDA
ABECDF	ACEBDF	ADEBCF	AEDBCF	AFDBCE	BAECDF	BCEADF	BDEACF	BEDACF	BFDAEC
ABECFD	ACEBFD	ADEBFC	AEDBFC	AFDBEC	BAECFD	BCEAFD	BDEAFC	BEDAFC	BFDAEC
ABEDCF	ACEDBF	ADECBF	AEDCBF	AFDCBE	BAEDCF	BCEDAF	BDECAF	BEDCAF	BFDCAE
ABEDFC	ACEDFB	ADECFB	AEDCFB	AFDCEB	BAEDFC	BCEDFA	BDECF A	BEDCF A	BFDC EA
ABE FCD	ACE FBD	ADE FBC	AED FBC	AFDEBC	BAE FCD	BCE FAD	BDE FAC	BED FAC	BFDEAC
ABE FDC	ACE FDB	ADE FCB	AED FCB	AFDECB	BAE FDC	BCE FDA	BDE FCA	BED FCA	BFDECA
ABFCDE	ACFBDE	ADFBCE	AEFBCD	AFEB CD	BAFCDE	BCFAED	BDFACE	BEFACD	BFEACD
ABFCED	ACFBED	ADFBEC	AEFBDC	AFEBDC	BAFCED	BCFAED	BDFACE	BEFADC	BFEADC
ABFDCE	ACFD BE	ADFCBE	AEF CBD	AFECBD	BAFDCE	BCFDAE	BDFCAE	BEFCAD	BFECAD
ABFDEC	ACFDEB	ADFCEB	AEF CDB	AFECDB	BAFDEC	BCFDEA	BDFCEA	BEFCDA	BFECDA
ABFECD	ACFEBD	ADFECB	AEFDBC	AFEDBC	BAFECD	BCFEAD	BDFEAC	BEFDAC	BFEDAC
ABFEDC	ACFEDB	ADFECB	AEFDCB	AFEDCB	BAFEDC	BCFEDA	BDFECA	BEFDCA	BFEDCA

Six players (part 1 of 3 parts)

# Six Players

CABDEF	CBADFE	CDABEF	CEABDF	CFABDE	DABCEF	DBACEF	DCABEF	DEABCF	DFABCE
CABDFE	CBADF	CDABFE	CEABFD	CFABED	DABCFE	DBACFE	DCABFE	DEABFC	DFABEC
CABEDF	CBAEDF	CDAEBF	CEADBF	CFADBE	DABECF	DBAECF	DCAEBF	DEACBF	DFACBE
CABEFD	CBAEFD	CDAEFB	CEADFB	CFADEB	DABEFC	DBAEFC	DCAEFB	DEACFB	DFACEB
CABFDE	CBAFDE	CDAFBE	CEAFBD	CFAEBD	DABFCE	DBAFCE	DCAFBE	DEAFBC	DFAEBC
CABFED	CBAFED	CDAFEB	CEAFDB	CFAEDB	DABFEC	DBAFEC	DCAFEB	DEAFCB	DFAECB
CADBFE	CBDAEF	CDBAEF	CEBADF	CFBAED	DACBEF	DBCAEF	DCBAEF	DEBACF	DFBACE
CADBFE	CBDAFE	CDBAFE	CEBAFD	CFBAED	DACBFE	DBCAGE	DCBAFE	DEBAFC	DFBAEC
CADEBF	CBDEAF	CDBEAF	CEBDAF	CFBDAE	DACEBF	DBCEAF	DCBEAF	DEBCAF	DFBCAE
CADEFB	CBDEFA	CDBEFA	CEBDFA	CFBDEA	DACEFB	DBCEFA	DCBEFA	DEBCFA	DFBCEA
CADFBE	CBDFAE	CDBFAE	CEBFAD	CFBEAD	DACFBE	DBCFAE	DCBFAE	DEBFAC	DFBEAC
CADFEB	CBDFEA	CDBFEA	CEBFDA	CFBEDA	DACFEB	DBCFEA	DCBFEA	DEBFCA	DFBECA
CAEBDF	CBEADF	CDEABF	CEDABF	CFDABE	DAEBCF	DBEACF	DCEABF	DECBAB	DFCABE
CAEBFD	CBEAFD	CDEAFB	CEDAFB	CFDAEB	DAEBFC	DBEAFC	DCEAFB	DECAFB	DFCAEB
CAEDBF	CBEDAF	CDEBAF	CEDBAF	CFDBAE	DAECBF	DBECAF	DCEBAF	DECBAF	DFCBAE
CAEDFB	CBEDFA	CDEBFA	CEDBFA	CFDBEA	DAECFB	DBECFA	DCEBFA	DECBFA	DFCBEA
CAEFBD	CBEFAD	CDEFAB	CEDFAB	CFDEAB	DAEFBC	DBEFAC	DCEFAB	DECFAB	DFCEAB
CAEFDB	CBEFDA	CDEFBA	CEDFBA	CFDEBA	DAEFCB	DBEFCA	DCEFBA	DECFBA	DFCEBA
CAFBD	CBFAED	CDFAEB	CEFABD	CFEABD	DAFBCE	DBFACE	DCFBAE	DEFABC	DFEABC
CAFBED	CBFAED	CDFAEB	CEFADB	CFEADB	DAFBEC	DBFAEC	DCFABE	DEFACB	DFEACB
CAFDBE	CBFDAE	CDFBAE	CEFBAD	CFEBAD	DAFCBE	DBFCAE	DCFBAE	DEFBAC	DFEBAC
CAFDEB	CBFDEA	CDFBEA	CEFBDA	CFEBDA	DAFCEB	DBFCEA	DCFBEA	DEFBCA	DFEBCA
CAFEBD	CBFEAD	CDFEAB	CEFDAB	CFEDAB	DAFEBC	DBFEAC	DCFEAB	DEFCAB	DFECAB
CAFEDB	CBFEDA	CDFEBA	CEFDAB	CFEDBA	DAFECB	DBFECA	DCFEB	DEFCBA	DFECBA

Six players (part 2 of 3 parts)

# Six Players

EABCFD	EBACDF	ECABDF	EDABCF	EFABCD	FABCDE	FBCADE	FCABDE	FDABCE	FEABCD
EABCFD	EBACFD	ECABFD	EDABFC	EFABDC	FABCED	FBACED	FCABED	FDABEC	FEABDC
EABDCF	EBADCF	ECADBF	EDACBF	EFACBD	FABDCE	FBADCE	FCADBE	FDACBE	FEACBD
EABDFC	EBADFC	ECADFB	EDACFB	EFACDB	FABDEC	FBADEC	FCADEB	FDACEB	FEACDB
EABFCD	EBAFCD	ECAFBD	EDAFBC	EFADBC	FABECD	FBAECD	CAEBCD	FDAEBC	FEADBC
EABFDC	EBAFDC	ECAFDB	EDAFCB	EFADCB	FABEDC	FBAEDC	CAEEDB	FDAECB	FEADCB
EACBDF	EBCADF	ECBADF	EDBACF	EFBADC	FACBDE	FBCADE	FCBADE	FDBACE	FEBACD
EACBFD	EBCAFD	ECBAFD	EDBAFC	EFBADC	FACBED	FBCAED	FCBAED	FDBAEC	FEBADC
EACDBF	EBCDAF	ECBDAF	EDBCAF	EFBCAD	FACDBE	FBCDAE	FCBDAE	FDBCAE	FEBCAD
EACDFB	EBCDFA	ECBDFA	EDBCFA	EFBCDA	FACDEB	FBCDEA	FCBDEA	FDBCEA	FEBFDA
EACFBD	EBCFAD	ECBFAD	EDBFAC	EFBDAC	FACEBD	FBCEAD	FCBEAD	FDBEAC	FEBDAC
EACFDB	EBCFDA	ECBFDA	EDBFCA	EFBDCA	FACEDB	FBCEAD	FCBEDA	FDBECA	FEBFDA
EADBCF	EBDACF	ECDABF	EDCABF	EFCABD	FADBCE	FBDAEC	FCDAEB	FDCABE	FECABD
EADBFC	EBDAFC	ECDAFB	EDCAFB	EFCADB	FADBEC	FBDAEC	FCDAEB	FDCAEB	FECADB
EADCBF	EBDCAF	ECDBAF	EDCBAF	EFCBAD	FADCBE	FBDAEC	FCDBAE	FDCBAE	FECBAD
EADCFB	EBDCFA	ECDBFA	EDCBFA	EFCBDA	FADCEB	FBDCFA	FCDBEA	FDCBEA	FECBDA
EADFBC	EBDFAC	ECDFAB	EDCFAB	EFCDAB	FADEBC	FBDEAC	FCDEAB	FDCEAB	FECDAB
EADFCB	EBDFCA	ECDFBA	EDCFBA	EFCDBA	FADECB	FBDECA	FCDEBA	FDCEBA	FECDBA
EAFBCD	EBFACD	ECFABD	EDFABC	EFDABC	FAEBDC	FBEACD	FCEABD	FDEABC	FEDABC
EAFBDC	EBFADC	ECFADB	EDFACB	EFDACB	FAEBDC	FBEADC	FCEADB	FDEACB	FEDACB
EAF CBD	EBFCAD	ECFBAD	EDFBAC	EFDBAC	FAECBD	FBECAD	FCEBAD	FDEBAC	FEDBAC
EAF CDB	EBFCDA	ECFBDA	EDFBCA	EFDBCA	FAECDB	FBECDA	FCEBDA	FDEBCA	FEDBCA
EAFDBC	EBFDAC	ECFDAB	EDFCAB	EFDCAB	FAEDBC	FBEDAC	FCEDAB	FDECAB	FEDCAB
EAFDCB	EBFDCA	ECFDBA	EDFCBA	EFDCBA	FAEDCB	FBEDCA	FCEDBA	FDECBA	FEDCBA

Six players (part 3 of 3 parts)

# Seven Players

- Click [here](#) for seven players.

# Outline

- 1 Introduction
- 2 Definitions
- 3 Listing Permutations
- 4 Pairs vs. Coalitions vs. Sequential Coalitions**
- 5 Shapley-Shubik Power
- 6 Examples
- 7 The Electoral College
- 8 Assignment



# Pairs vs. Coalitions vs. Sequential Coalitions

## Pairs vs. Coalitions vs. Sequential Coalitions

$N$	Pairs (Condorcet)	Coalitions (Banzhaf)	Sequential Coalitions (Shapley)
1	0	2	1
2			
3			
4			
5			
6			
7			
8			
9			
10			

# Pairs vs. Coalitions vs. Sequential Coalitions

## Pairs vs. Coalitions vs. Sequential Coalitions

$N$	Pairs (Condorcet)	Coalitions (Banzhaf)	Sequential Coalitions (Shapley)
1	0	2	1
2	1	4	2
3			
4			
5			
6			
7			
8			
9			
10			

# Pairs vs. Coalitions vs. Sequential Coalitions

## Pairs vs. Coalitions vs. Sequential Coalitions

$N$	Pairs (Condorcet)	Coalitions (Banzhaf)	Sequential Coalitions (Shapley)
1	0	2	1
2	1	4	2
3	3	8	6
4			
5			
6			
7			
8			
9			
10			

# Pairs vs. Coalitions vs. Sequential Coalitions

## Pairs vs. Coalitions vs. Sequential Coalitions

$N$	Pairs (Condorcet)	Coalitions (Banzhaf)	Sequential Coalitions (Shapley)
1	0	2	1
2	1	4	2
3	3	8	6
4	6	16	24
5			
6			
7			
8			
9			
10			

# Pairs vs. Coalitions vs. Sequential Coalitions

## Pairs vs. Coalitions vs. Sequential Coalitions

$N$	Pairs (Condorcet)	Coalitions (Banzhaf)	Sequential Coalitions (Shapley)
1	0	2	1
2	1	4	2
3	3	8	6
4	6	16	24
5	10	32	120
6			
7			
8			
9			
10			

# Pairs vs. Coalitions vs. Sequential Coalitions

## Pairs vs. Coalitions vs. Sequential Coalitions

$N$	Pairs (Condorcet)	Coalitions (Banzhaf)	Sequential Coalitions (Shapley)
1	0	2	1
2	1	4	2
3	3	8	6
4	6	16	24
5	10	32	120
6	15	64	720
7			
8			
9			
10			

# Pairs vs. Coalitions vs. Sequential Coalitions

## Pairs vs. Coalitions vs. Sequential Coalitions

$N$	Pairs (Condorcet)	Coalitions (Banzhaf)	Sequential Coalitions (Shapley)
1	0	2	1
2	1	4	2
3	3	8	6
4	6	16	24
5	10	32	120
6	15	64	720
7	21	128	5,040
8			
9			
10			

# Pairs vs. Coalitions vs. Sequential Coalitions

## Pairs vs. Coalitions vs. Sequential Coalitions

$N$	Pairs (Condorcet)	Coalitions (Banzhaf)	Sequential Coalitions (Shapley)
1	0	2	1
2	1	4	2
3	3	8	6
4	6	16	24
5	10	32	120
6	15	64	720
7	21	128	5,040
8	28	256	40,320
9			
10			



# Pairs vs. Coalitions vs. Sequential Coalitions

## Pairs vs. Coalitions vs. Sequential Coalitions

$N$	Pairs (Condorcet)	Coalitions (Banzhaf)	Sequential Coalitions (Shapley)
1	0	2	1
2	1	4	2
3	3	8	6
4	6	16	24
5	10	32	120
6	15	64	720
7	21	128	5,040
8	28	256	40,320
9	36	512	362,880
10			

# Pairs vs. Coalitions vs. Sequential Coalitions

## Pairs vs. Coalitions vs. Sequential Coalitions

$N$	Pairs (Condorcet)	Coalitions (Banzhaf)	Sequential Coalitions (Shapley)
1	0	2	1
2	1	4	2
3	3	8	6
4	6	16	24
5	10	32	120
6	15	64	720
7	21	128	5,040
8	28	256	40,320
9	36	512	362,880
10	45	1,024	3,628,800

# Outline

- 1 Introduction
- 2 Definitions
- 3 Listing Permutations
- 4 Pairs vs. Coalitions vs. Sequential Coalitions
- 5 Shapley-Shubik Power**
- 6 Examples
- 7 The Electoral College
- 8 Assignment

# Example

## Example

Consider the situation  $[5 : 4, 2, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Sequential Coalition	Vote Totals		
	1st	2nd	3rd
$\{A, B, C\}$			
$\{A, C, B\}$			
$\{B, A, C\}$			
$\{B, C, A\}$			
$\{C, A, B\}$			
$\{C, B, A\}$			

# Example

## Example

Consider the situation  $[5 : 4, 2, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Sequential Coalition	Vote Totals		
	1st	2nd	3rd
$\{A, B, C\}$	4		
$\{A, C, B\}$	4		
$\{B, A, C\}$	2		
$\{B, C, A\}$	2		
$\{C, A, B\}$	1		
$\{C, B, A\}$	1		

# Example

## Example

Consider the situation  $[5 : 4, 2, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Sequential Coalition	Vote Totals		
	1st	2nd	3rd
$\{A, B, C\}$	4	6	
$\{A, C, B\}$	4	5	
$\{B, A, C\}$	2	6	
$\{B, C, A\}$	2	3	
$\{C, A, B\}$	1	5	
$\{C, B, A\}$	1	3	

# Example

## Example

Consider the situation  $[5 : 4, 2, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Sequential Coalition	Vote Totals		
	1st	2nd	3rd
$\{A, B, C\}$	4	6	
$\{A, C, B\}$	4	5	
$\{B, A, C\}$	2	6	
$\{B, C, A\}$	2	3	
$\{C, A, B\}$	1	5	
$\{C, B, A\}$	1	3	

# Example

## Example

Consider the situation  $[5 : 4, 2, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Sequential Coalition	Vote Totals		
	1st	2nd	3rd
$\{A, B, C\}$	4	6	7
$\{A, C, B\}$	4	5	7
$\{B, A, C\}$	2	6	7
$\{B, C, A\}$	2	3	7
$\{C, A, B\}$	1	5	7
$\{C, B, A\}$	1	3	7



# Example

## Example

Consider the situation  $[5 : 4, 2, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Sequential Coalition	Vote Totals		
	1st	2nd	3rd
$\{A, B, C\}$	4	6	7
$\{A, C, B\}$	4	5	7
$\{B, A, C\}$	2	6	7
$\{B, C, A\}$	2	3	7
$\{C, A, B\}$	1	5	7
$\{C, B, A\}$	1	3	7

# Shapley-Shubik Power

## Definition (Pivotal Count)

A player's **pivotal count** is the number of sequential coalitions in which he is the pivotal player.

In the previous example, the pivotal counts are 4, 1, 1.

## Definition (Shapley-Shubik Power Index)

The **Shapley-Shubik power index (SSPI)** for a player is that player's pivotal count divided by the total of the counts.

## Definition (Shapley-Shubik Power Distribution)

The **Shapley-Shubik power distribution** is the set of SSPI's for all the players.

# Outline

- 1 Introduction
- 2 Definitions
- 3 Listing Permutations
- 4 Pairs vs. Coalitions vs. Sequential Coalitions
- 5 Shapley-Shubik Power
- 6 Examples**
- 7 The Electoral College
- 8 Assignment

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>ABCD</i>				
<i>ABDC</i>				
<i>ACBD</i>				
<i>ACDB</i>				
<i>ADBC</i>				
<i>ADCB</i>				

Seq. Coal.	1st	2nd	3rd	4th
<i>BACD</i>				
<i>BADC</i>				
<i>BCAD</i>				
<i>BCDA</i>				
<i>BDAC</i>				
<i>BDCA</i>				

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>ABCD</i>	9			
<i>ABDC</i>	9			
<i>ACBD</i>	9			
<i>ACDB</i>	9			
<i>ADBC</i>	9			
<i>ADCB</i>	9			

Seq. Coal.	1st	2nd	3rd	4th
<i>BACD</i>				
<i>BADC</i>				
<i>BCAD</i>				
<i>BCDA</i>				
<i>BDAC</i>				
<i>BDCA</i>				

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>ABCD</i>	9	17		
<i>ABDC</i>	9	17		
<i>ACBD</i>	9	12		
<i>ACDB</i>	9	12		
<i>ADBC</i>	9	10		
<i>ADCB</i>	9	10		

Seq. Coal.	1st	2nd	3rd	4th
<i>BACD</i>				
<i>BADC</i>				
<i>BCAD</i>				
<i>BCDA</i>				
<i>BDAC</i>				
<i>BDCA</i>				

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>ABCD</i>	9	17		
<i>ABDC</i>	9	17		
<i>ACBD</i>	9	12		
<i>ACDB</i>	9	12		
<i>ADBC</i>	9	10		
<i>ADCB</i>	9	10		

Seq. Coal.	1st	2nd	3rd	4th
<i>BACD</i>				
<i>BADC</i>				
<i>BCAD</i>				
<i>BCDA</i>				
<i>BDAC</i>				
<i>BDCA</i>				

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>ABCD</i>	9	17	20	
<i>ABDC</i>	9	17	18	
<i>ACBD</i>	9	12	20	
<i>ACDB</i>	9	12	13	
<i>ADBC</i>	9	10	18	
<i>ADCB</i>	9	10	13	

Seq. Coal.	1st	2nd	3rd	4th
<i>BACD</i>				
<i>BADC</i>				
<i>BCAD</i>				
<i>BCDA</i>				
<i>BDAC</i>				
<i>BDCA</i>				



# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>ABCD</i>	9	17	20	
<i>ABDC</i>	9	17	18	
<i>ACBD</i>	9	12	20	
<i>ACDB</i>	9	12	13	
<i>ADBC</i>	9	10	18	
<i>ADCB</i>	9	10	13	

Seq. Coal.	1st	2nd	3rd	4th
<i>BACD</i>				
<i>BADC</i>				
<i>BCAD</i>				
<i>BCDA</i>				
<i>BDAC</i>				
<i>BDCA</i>				

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>ABCD</i>	9	17	20	21
<i>ABDC</i>	9	17	18	21
<i>ACBD</i>	9	12	20	21
<i>ACDB</i>	9	12	13	21
<i>ADBC</i>	9	10	18	21
<i>ADCB</i>	9	10	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>BACD</i>				
<i>BADC</i>				
<i>BCAD</i>				
<i>BCDA</i>				
<i>BDAC</i>				
<i>BDCA</i>				

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>ABCD</i>	9	17	20	21
<i>ABDC</i>	9	17	18	21
<i>ACBD</i>	9	12	20	21
<i>ACDB</i>	9	12	13	21
<i>ADBC</i>	9	10	18	21
<i>ADCB</i>	9	10	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>BACD</i>	8			
<i>BADC</i>	8			
<i>BCAD</i>	8			
<i>BCDA</i>	8			
<i>BDAC</i>	8			
<i>BDCA</i>	8			

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>ABCD</i>	9	17	20	21
<i>ABDC</i>	9	17	18	21
<i>ACBD</i>	9	12	20	21
<i>ACDB</i>	9	12	13	21
<i>ADBC</i>	9	10	18	21
<i>ADCB</i>	9	10	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>BACD</i>	8	17		
<i>BADC</i>	8	17		
<i>BCAD</i>	8	11		
<i>BCDA</i>	8	11		
<i>BDAC</i>	8	9		
<i>BDCA</i>	8	9		

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>ABCD</i>	9	17	20	21
<i>ABDC</i>	9	17	18	21
<i>ACBD</i>	9	12	20	21
<i>ACDB</i>	9	12	13	21
<i>ADBC</i>	9	10	18	21
<i>ADCB</i>	9	10	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>BACD</i>	8	17		
<i>BADC</i>	8	17		
<i>BCAD</i>	8	11		
<i>BCDA</i>	8	11		
<i>BDAC</i>	8	9		
<i>BDCA</i>	8	9		

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>ABCD</i>	9	17	20	21
<i>ABDC</i>	9	17	18	21
<i>ACBD</i>	9	12	20	21
<i>ACDB</i>	9	12	13	21
<i>ADBC</i>	9	10	18	21
<i>ADCB</i>	9	10	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>BACD</i>	8	17	20	
<i>BADC</i>	8	17	18	
<i>BCAD</i>	8	11	20	
<i>BCDA</i>	8	11	12	
<i>BDAC</i>	8	9	18	
<i>BDCA</i>	8	9	12	

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>ABCD</i>	9	17	20	21
<i>ABDC</i>	9	17	18	21
<i>ACBD</i>	9	12	20	21
<i>ACDB</i>	9	12	13	21
<i>ADBC</i>	9	10	18	21
<i>ADCB</i>	9	10	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>BACD</i>	8	17	20	
<i>BADC</i>	8	17	18	
<i>BCAD</i>	8	11	20	
<i>BCDA</i>	8	11	12	
<i>BDAC</i>	8	9	18	
<i>BDCA</i>	8	9	12	

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>ABCD</i>	9	17	20	21
<i>ABDC</i>	9	17	18	21
<i>ACBD</i>	9	12	20	21
<i>ACDB</i>	9	12	13	21
<i>ADBC</i>	9	10	18	21
<i>ADCB</i>	9	10	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>BACD</i>	8	17	20	21
<i>BADC</i>	8	17	18	21
<i>BCAD</i>	8	11	20	21
<i>BCDA</i>	8	11	12	21
<i>BDAC</i>	8	9	18	21
<i>BDCA</i>	8	9	12	21



# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>CABD</i>				
<i>CADB</i>				
<i>CBAD</i>				
<i>CBDA</i>				
<i>CDAB</i>				
<i>CDBA</i>				

Seq. Coal.	1st	2nd	3rd	4th
<i>DABC</i>				
<i>DBC B</i>				
<i>DBAC</i>				
<i>DBCA</i>				
<i>DCAB</i>				
<i>DCBA</i>				

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>CABD</i>	3			
<i>CADB</i>	3			
<i>CBAD</i>	3			
<i>CBDA</i>	3			
<i>CDAB</i>	3			
<i>CDBA</i>	3			

Seq. Coal.	1st	2nd	3rd	4th
<i>DABC</i>				
<i>DBCB</i>				
<i>DBAC</i>				
<i>DBCA</i>				
<i>DCAB</i>				
<i>DCBA</i>				

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>CABD</i>	3	12		
<i>CADB</i>	3	12		
<i>CBAD</i>	3	11		
<i>CBDA</i>	3	11		
<i>CDAB</i>	3	4		
<i>CDBA</i>	3	4		

Seq. Coal.	1st	2nd	3rd	4th
<i>DABC</i>				
<i>DBCB</i>				
<i>DBAC</i>				
<i>DBCA</i>				
<i>DCAB</i>				
<i>DCBA</i>				

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>CABD</i>	3	12		
<i>CADB</i>	3	12		
<i>CBAD</i>	3	11		
<i>CBDA</i>	3	11		
<i>CDAB</i>	3	4		
<i>CDBA</i>	3	4		

Seq. Coal.	1st	2nd	3rd	4th
<i>DABC</i>				
<i>DBCB</i>				
<i>DBAC</i>				
<i>DBCA</i>				
<i>DCAB</i>				
<i>DCBA</i>				

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>CABD</i>	3	12	20	
<i>CADB</i>	3	12	13	
<i>CBAD</i>	3	11	20	
<i>CBDA</i>	3	11	12	
<i>CDAB</i>	3	4	13	
<i>CDBA</i>	3	4	13	

Seq. Coal.	1st	2nd	3rd	4th
<i>DABC</i>				
<i>DBCB</i>				
<i>DBAC</i>				
<i>DBCA</i>				
<i>DCAB</i>				
<i>DCBA</i>				

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>CABD</i>	3	12	20	
<i>CADB</i>	3	12	13	
<i>CBAD</i>	3	11	20	
<i>CBDA</i>	3	11	12	
<i>CDAB</i>	3	4	13	
<i>CDBA</i>	3	4	13	

Seq. Coal.	1st	2nd	3rd	4th
<i>DABC</i>				
<i>DBCB</i>				
<i>DBAC</i>				
<i>DBCA</i>				
<i>DCAB</i>				
<i>DCBA</i>				

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>C</i> <b>A</b> <i>BD</i>	3	12	20	21
<i>C</i> <b>A</b> <i>DB</i>	3	12	13	21
<i>C</i> <b>B<i>AD</i></b>	3	11	20	21
<i>C</i> <b>B<i>DA</i></b>	3	11	12	21
<i>C</i> <b>D<i>AB</i></b>	3	4	13	21
<i>C</i> <b>D<i>BA</i></b>	3	4	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>D</i> <i>A</i> <i>B</i> <i>C</i>				
<i>D</i> <i>B</i> <i>C</i> <i>B</i>				
<i>D</i> <i>B</i> <i>A</i> <i>C</i>				
<i>D</i> <i>B</i> <i>C</i> <i>A</i>				
<i>D</i> <i>C</i> <i>A</i> <i>B</i>				
<i>D</i> <i>C</i> <i>B</i> <i>A</i>				

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>C</i> <b>A</b> <i>BD</i>	3	12	20	21
<i>C</i> <b>A</b> <i>DB</i>	3	12	13	21
<i>C</i> <b>B<i>AD</i></b>	3	11	20	21
<i>C</i> <b>B<i>DA</i></b>	3	11	12	21
<i>C</i> <b>D<i>AB</i></b>	3	4	13	21
<i>C</i> <b>D<i>BA</i></b>	3	4	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>D</i> <b>A</b> <i>BC</i>	1			
<i>D</i> <b>B</b> <i>CB</i>	1			
<i>D</i> <b>B</b> <i>AC</i>	1			
<i>D</i> <b>B</b> <i>CA</i>	1			
<i>D</i> <b>C</b> <i>AB</i>	1			
<i>D</i> <b>C</b> <i>BA</i>	1			



# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>C</i> <b>A</b> <i>BD</i>	3	12	20	21
<i>C</i> <b>A</b> <i>DB</i>	3	12	13	21
<i>C</i> <b>B<i>AD</i></b>	3	11	20	21
<i>C</i> <b>B<i>DA</i></b>	3	11	12	21
<i>C</i> <b>D<i>AB</i></b>	3	4	13	21
<i>C</i> <b>D<i>BA</i></b>	3	4	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>D</i> <b>A</b> <i>BC</i>	1	10		
<i>D</i> <b>B</b> <i>CB</i>	1	10		
<i>D</i> <b>B</b> <i>AC</i>	1	9		
<i>D</i> <b>B</b> <i>CA</i>	1	9		
<i>D</i> <b>C</b> <i>AB</i>	1	4		
<i>D</i> <b>C</b> <i>BA</i>	1	4		

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>C</i> <b>A</b> <i>BD</i>	3	12	20	21
<i>C</i> <b>A</b> <i>DB</i>	3	12	13	21
<i>C</i> <b>B<i>AD</i></b>	3	11	20	21
<i>C</i> <b>B<i>DA</i></b>	3	11	12	21
<i>C</i> <b>D<i>AB</i></b>	3	4	13	21
<i>C</i> <b>D<i>BA</i></b>	3	4	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>D</i> <b>A</b> <i>BC</i>	1	10	18	
<i>D</i> <b>B</b> <i>CB</i>	1	10	13	
<i>D</i> <b>B</b> <i>AC</i>	1	9	18	
<i>D</i> <b>B</b> <i>CA</i>	1	9	12	
<i>D</i> <b>C</b> <i>AB</i>	1	4	13	
<i>D</i> <b>C</b> <i>BA</i>	1	4	12	

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>C</i> <b>A</b> <i>BD</i>	3	12	20	21
<i>C</i> <b>A</b> <i>DB</i>	3	12	13	21
<i>C</i> <b>B<i>AD</i></b>	3	11	20	21
<i>C</i> <b>B<i>DA</i></b>	3	11	12	21
<i>C</i> <b>D<i>AB</i></b>	3	4	13	21
<i>C</i> <b>D<i>BA</i></b>	3	4	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>D</i> <b>A</b> <i>BC</i>	1	10	18	
<i>D</i> <b>B</b> <i>CB</i>	1	10	13	
<i>D</i> <b>B</b> <i>AC</i>	1	9	18	
<i>D</i> <b>B</b> <i>CA</i>	1	9	12	
<i>D</i> <b>C</b> <i>AB</i>	1	4	13	
<i>D</i> <b>C</b> <i>BA</i>	1	4	12	

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>C</i> <b>A</b> <i>BD</i>	3	12	20	21
<i>C</i> <b>A</b> <i>DB</i>	3	12	13	21
<i>C</i> <b>B<i>AD</i></b>	3	11	20	21
<i>C</i> <b>B<i>DA</i></b>	3	11	12	21
<i>C</i> <b>D<i>AB</i></b>	3	4	13	21
<i>C</i> <b>D<i>BA</i></b>	3	4	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>D</i> <b>A</b> <i>BC</i>	1	10	18	21
<i>D</i> <b>B<i>CB</i></b>	1	10	13	21
<i>D</i> <b>B<i>AC</i></b>	1	9	18	21
<i>D</i> <b>B<i>CA</i></b>	1	9	12	21
<i>D</i> <b>C<i>AB</i></b>	1	4	13	21
<i>D</i> <b>C<i>BA</i></b>	1	4	12	21

# Example

## Example

Consider the situation  $[11 : 9, 8, 3, 1]$ . List all sequential coalitions and determine the pivotal player for each one.

Seq. Coal.	1st	2nd	3rd	4th
<i>C</i> <b>A</b> <i>BD</i>	3	12	20	21
<i>C</i> <b>A</b> <i>DB</i>	3	12	13	21
<i>C</i> <b>B<i>AD</i></b>	3	11	20	21
<i>C</i> <b>B<i>DA</i></b>	3	11	12	21
<i>C</i> <b>D<i>AB</i></b>	3	4	13	21
<i>C</i> <b>D<i>BA</i></b>	3	4	13	21

Seq. Coal.	1st	2nd	3rd	4th
<i>D</i> <b>A</b> <i>BC</i>	1	10	18	21
<i>D</i> <b>B<i>CB</i></b>	1	10	13	21
<i>D</i> <b>B<i>AC</i></b>	1	9	18	21
<i>D</i> <b>B<i>CA</i></b>	1	9	12	21
<i>D</i> <b>C<i>AB</i></b>	1	4	13	21
<i>D</i> <b>C<i>BA</i></b>	1	4	12	21

# Example

## Example

- Find the Shapley-Shubik power distribution for  $[21 : 12, 11, 8, 6, 5]$ .
- Compare it to the Banzhaf power distribution.

# Example

## Example

- Find the Shapley-Shubik power distribution for  $[34 : 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1]$ .
- Compare it to the Banzhaf power distribution.

# Outline

- 1 Introduction
- 2 Definitions
- 3 Listing Permutations
- 4 Pairs vs. Coalitions vs. Sequential Coalitions
- 5 Shapley-Shubik Power
- 6 Examples
- 7 The Electoral College**
- 8 Assignment



# The Electoral College

- In the electoral college, each state get a number of votes equal to its number of representatives plus its number of senators.
- “Each State shall appoint, in such Manner as the Legislature thereof may direct, a Number of Electors, equal to the whole Number of Senators and Representatives to which the State may be entitled in the Congress.”
- The number of representatives is proportional to the state’s population, so why add the two senators?

# The Electoral College

- In the electoral college, each state get a number of votes equal to its number of representatives plus its number of senators.
- “Each State shall appoint, in such Manner as the Legislature thereof may direct, a Number of Electors, equal to the whole Number of Senators and Representatives to which the State may be entitled in the Congress.”
- The number of representatives is proportional to the state’s population, so why add the two senators?
- It was to give the smaller states more power relative to the larger states.

# The Election of 1788

- In 1788, only 11 states participated in the presidential election.
- They cast a total of 81 electoral votes, 41 needed to win.

VA	12
MA	10
PA	10
MD	8
NY	8
CT	7

SC	7
NJ	6
GA	5
NH	5
DE	3

- Determine the power of each state.

# The Election of 1788

- What if the senators were not counted?
- They would cast a total of 59 electoral votes, 30 needed to win.

VA	10
MA	8
PA	8
MD	6
NY	6
CT	5

SC	5
NJ	4
GA	3
NH	3
DE	1

- Determine the power of each state.

# The Election of 1792

- In 1792, 15 states participated in the presidential election.
- They cast a total of 135 electoral votes, 68 needed to win.

VA	21
MA	16
PA	15
NY	12
NC	12

MD	10
CT	9
SC	8
NJ	7
NH	6

GA	4
KY	4
RI	4
VT	4
DE	3

- Determine the power of each state.

# The Election of 1788

- What if the senators were not counted?
- They would cast a total of 105 electoral votes, 53 needed to win.

VA	19
MA	14
PA	13
NY	10
NC	10

MD	8
CT	7
SC	6
NJ	5
NH	4

GA	2
KY	2
RI	2
VT	2
DE	1

- Determine the power of each state.

# Outline

- 1 Introduction
- 2 Definitions
- 3 Listing Permutations
- 4 Pairs vs. Coalitions vs. Sequential Coalitions
- 5 Shapley-Shubik Power
- 6 Examples
- 7 The Electoral College
- 8 Assignment**

# Assignment

## Assignment

- Ch. 2: 27, 28, 29, 30, 32, 33; 56.