

The Method of Sealed Bids

Lecture 18
Section 3.5

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- 1 The Method of Sealed Bids
- 2 An Example – 3 Players, 2 Assets
 - The Bidding
 - The Allocation
 - The First Settlement
 - The Division of the Surplus
 - The Final Settlement
- 3 Another Example – 5 Players, 3 Assets
- 4 Another Example – 2 Players, 4 Assets
- 5 Assignment

1 The Method of Sealed Bids

2 An Example – 3 Players, 2 Assets

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3 Another Example – 5 Players, 3 Assets

4 Another Example – 2 Players, 4 Assets

5 Assignment

The Method of Sealed Bids

- The **method of sealed bids** is used to divide assets that cannot be subdivided.
- For example, if there are 2 assets to be divided 3 ways, then either
 - Two players get the two assets and the third player is compensated in cash, or
 - One player gets both assets and the other two players are compensated in cash.
- The first step is for each player to submit a sealed bid for each asset.

The Method of Sealed Bids

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- For example, if there are 2 assets to be divided 3 ways, then either
 - Two players get the two assets and the third player is compensated in cash, or
 - One player gets both assets and the other two players are compensated in cash.
- The first step is for each player to submit a sealed bid for each asset.
- This method is an **algorithm** (no judgment calls).

The Method of Sealed Bids

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- For example, if there are 2 assets to be divided 3 ways, then either
 - Two players get the two assets and the third player is compensated in cash, or
 - One player gets both assets and the other two players are compensated in cash.
- The first step is for each player to submit a sealed bid for each asset.
- This method is an **algorithm** (no judgment calls). **Yea!**

The Method of Sealed Bids

- The five steps (to be explained) are
 - The bidding
 - The allocation of assets
 - The first settlement
 - Dividing the surplus
 - The final settlement

Outline

1 The Method of Sealed Bids

2 An Example – 3 Players, 2 Assets

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The Method of Sealed Bids

Example (3 Players, 2 Assets)

- Three guys, Andy, Bob, and Chuck, are dividing two assets of their parents' estate.
- The assets are
 - The house
 - The yacht.

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The Method of Sealed Bids

The Bidding

- Each player submits a **bid** on each asset.
- No player knows any other player's bids.
- Each player's bids determine **his** fair share.

The Method of Sealed Bids

Example (3 Players, 2 Assets)

	House	Yacht	Total Value	Fair Share
Andy	100	200		
Bob	180	150		
Chuck	150	90		

- Each player bids on each asset.
- The table shows their bids (in \$1000s) and their fair shares.

The Method of Sealed Bids

Example (3 Players, 2 Assets)

	House	Yacht	Total Value	Fair Share
Andy	100	200	300	
Bob	180	150	330	
Chuck	150	90	240	

- Each player bids on each asset.
- The table shows their bids (in \$1000s) and their fair shares.

The Method of Sealed Bids

Example (3 Players, 2 Assets)

	House	Yacht	Total Value	Fair Share
Andy	100	200	300	100
Bob	180	150	330	110
Chuck	150	90	240	80

- Each player bids on each asset.
- The table shows their bids (in \$1000s) and their fair shares.

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The Method of Sealed Bids

Allocation of Assets

- The **allocation** of the assets is based on the bids.
- Each asset goes to the highest bidder for that asset.

The Method of Sealed Bids

Example (3 Players, 2 Assets)

- Bob gets
- Andy gets
- Chuck gets

The Method of Sealed Bids

Example (3 Players, 2 Assets)

- Bob gets **the house**, worth \$180,000 (to him).
- Andy gets
- Chuck gets

The Method of Sealed Bids

Example (3 Players, 2 Assets)

- Bob gets **the house**, worth \$180,000 (to him).
- Andy gets **the yacht**, worth \$200,000 (to him).
- Chuck gets

The Method of Sealed Bids

Example (3 Players, 2 Assets)

- Bob gets **the house**, worth \$180,000 (to him).
- Andy gets **the yacht**, worth \$200,000 (to him).
- Chuck gets **nothing** (yet).

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The Method of Sealed Bids

The First Settlement

- In the **first settlement**, the players pay or receive cash to compensate for getting too little or too much in the allocation of assets.

The Method of Sealed Bids

The First Settlement

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- If the assets won by a player **exceed** his fair share, then he **pays** the difference in cash to the **surplus**.

The Method of Sealed Bids

The First Settlement

- In the **first settlement**, the players pay or receive cash to compensate for getting too little or too much in the allocation of assets.
- If the assets won by a player **exceed** his fair share, then he **pays** the difference in cash to the **surplus**.
- If the assets won (if any) by a player **fall short** of his fair share, then he **receives** the difference in cash from the **surplus**.

The Method of Sealed Bids

Example (3 Players, 2 Assets)

- We use cash to even things up.
 - Bob's fair share is \$110,000 and he got the house (worth \$180,000), so he
 - Andy's fair share is \$100,000 and he got the yacht (worth \$200,000), so he
 - Chuck's fair share is \$80,000 and he got nothing, so he

The Method of Sealed Bids

Example (3 Players, 2 Assets)

- We use cash to even things up.
 - Bob's fair share is \$110,000 and he got the house (worth \$180,000), so he **pays \$70,000** into the surplus.
 - Andy's fair share is \$100,000 and he got the yacht (worth \$200,000), so he
 - Chuck's fair share is \$80,000 and he got nothing, so he

The Method of Sealed Bids

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 - Chuck's fair share is \$80,000 and he got nothing, so he

The Method of Sealed Bids

Example (3 Players, 2 Assets)

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 - Bob's fair share is \$110,000 and he got the house (worth \$180,000), so he **pays \$70,000** into the surplus.
 - Andy's fair share is \$100,000 and he got the yacht (worth \$200,000), so he **pays \$100,000** into the surplus.
 - Chuck's fair share is \$80,000 and he got nothing, so he **receives \$80,000** from the surplus

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The Method of Sealed Bids

Dividing the Surplus

- Whatever is left over is the **surplus**.
- The surplus is **divided equally** among all the players.

The Method of Sealed Bids

Example (3 Players, 2 Assets)

- Bob paid \$70,000, Andy paid \$100,000, and Chuck received \$80.
- So the **surplus** is

The Method of Sealed Bids

Example (3 Players, 2 Assets)

- Bob paid \$70,000, Andy paid \$100,000, and Chuck received \$80.
- So the **surplus** is **\$90,000**.

The Method of Sealed Bids

Example (3 Players, 2 Assets)

- Bob paid \$70,000, Andy paid \$100,000, and Chuck received \$80.
- So the **surplus** is **\$90,000**.
- We divide it equally among the players, with each receiving

The Method of Sealed Bids

Example (3 Players, 2 Assets)

- Bob paid \$70,000, Andy paid \$100,000, and Chuck received \$80.
- So the **surplus** is **\$90,000**.
- We divide it equally among the players, with each receiving **\$30,000**.

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The Method of Sealed Bids

The Final Settlement

- The **final settlement** is a summary that describes for each player
 - Which assets he won (if any).
 - The **net** amount of cash that he paid or received.

The Method of Sealed Bids

Example (3 Players, 2 Assets)

- The final settlement is as follows:
 - Andy gets **the yacht** and **pays \$70,000** in cash.
 - Bob gets **the house** and **pays \$40,000** in cash.
 - Chuck gets **\$110,000** in cash.

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The Method of Sealed Bids

Example (3 Players, 2 Assets)

- Five children, Andy, Bob, Chuck, Dave, and Eddie are dividing three old camping items.
- The assets are
 - A tent
 - A stove
 - A backpack

The Method of Sealed Bids

Example (The Bidding)

	Tent	Stove	Backpack	Total Value	Fair Share
Andy	80	20	50		
Bob	120	30	40		
Chuck	100	35	45		
Dave	65	30	80		
Eddie	150	20	60		

The Bids

The Method of Sealed Bids

Example (The Bidding)

	Tent	Stove	Backpack	Total Value	Fair Share
Andy	80	20	50	150	
Bob	120	30	40	190	
Chuck	100	35	45	180	
Dave	65	30	80	175	
Eddie	150	20	60	230	

The Bids

The Method of Sealed Bids

Example (The Bidding)

	Tent	Stove	Backpack	Total Value	Fair Share
Andy	80	20	50	150	30
Bob	120	30	40	190	38
Chuck	100	35	45	180	36
Dave	65	30	80	175	35
Eddie	150	20	60	230	46

The Bids

The Method of Sealed Bids

Example (The Allocation of Assets)

- Andy gets
- Bob gets
- Chuck gets
- Dave gets
- Eddie gets

The Method of Sealed Bids

Example (The Allocation of Assets)

- Andy gets
- Bob gets
- Chuck gets
- Dave gets
- Eddie gets **the tent**, worth \$150.

The Method of Sealed Bids

Example (The Allocation of Assets)

- Andy gets
- Bob gets
- Chuck gets **the stove**, worth \$35.
- Dave gets
- Eddie gets **the tent**, worth \$150.

The Method of Sealed Bids

Example (The Allocation of Assets)

- Andy gets
- Bob gets
- Chuck gets **the stove**, worth \$35.
- Dave gets **the backpack**, worth \$80.
- Eddie gets **the tent**, worth \$150.

The Method of Sealed Bids

Example (The Allocation of Assets)

- Andy gets **nothing**, yet.
- Bob gets **nothing**, yet.
- Chuck gets **the stove**, worth \$35.
- Dave gets **the backpack**, worth \$80.
- Eddie gets **the tent**, worth \$150.

The Method of Sealed Bids

Example (The First Settlement)

- Andy
- Bob
- Chuck
- Dave
- Eddie

The Method of Sealed Bids

Example (The First Settlement)

- Andy receives \$30 in cash.
- Bob
- Chuck
- Dave
- Eddie

The Method of Sealed Bids

Example (The First Settlement)

- Andy **receives \$30** in cash.
- Bob **receives \$38** in cash.
- Chuck
- Dave
- Eddie

The Method of Sealed Bids

Example (The First Settlement)

- Andy receives \$30 in cash.
- Bob receives \$38 in cash.
- Chuck receives \$1 in cash.
- Dave
- Eddie

The Method of Sealed Bids

Example (The First Settlement)

- Andy **receives \$30** in cash.
- Bob **receives \$38** in cash.
- Chuck **receives \$1** in cash.
- Dave **pays \$45** in cash.
- Eddie

The Method of Sealed Bids

Example (The First Settlement)

- Andy **receives \$30** in cash.
- Bob **receives \$38** in cash.
- Chuck **receives \$1** in cash.
- Dave **pays \$45** in cash.
- Eddie **pays \$104** in cash.

The Method of Sealed Bids

Example (Dividing the Surplus)

- The surplus is

The Method of Sealed Bids

Example (Dividing the Surplus)

- The surplus is \$80.

The Method of Sealed Bids

Example (Dividing the Surplus)

- The surplus is \$80.
- So, each player gets

The Method of Sealed Bids

Example (Dividing the Surplus)

- The surplus is \$80.
- So, each player gets \$16.

Example (The Final Settlement)

- The final settlement is
 - Andy gets
 - Bob gets
 - Chuck gets
 - Dave gets
 - Eddie gets

The Method of Sealed Bids

Example (The Final Settlement)

- The final settlement is
 - Andy gets \$46 in cash.
 - Bob gets
 - Chuck gets
 - Dave gets
 - Eddie gets

The Method of Sealed Bids

Example (The Final Settlement)

- The final settlement is
 - Andy gets \$46 in cash.
 - Bob gets \$54 in cash.
 - Chuck gets
 - Dave gets
 - Eddie gets

Example (The Final Settlement)

- The final settlement is
 - Andy gets \$46 in cash.
 - Bob gets \$54 in cash.
 - Chuck gets the stove and gets \$17 in cash.
 - Dave gets
 - Eddie gets

The Method of Sealed Bids

Example (The Final Settlement)

- The final settlement is
 - Andy gets \$46 in cash.
 - Bob gets \$54 in cash.
 - Chuck gets the stove and gets \$17 in cash.
 - Dave gets the backpack and pays \$29 in cash.
 - Eddie gets

The Method of Sealed Bids

Example (The Final Settlement)

- The final settlement is
 - Andy gets \$46 in cash.
 - Bob gets \$54 in cash.
 - Chuck gets the stove and gets \$17 in cash.
 - Dave gets the backpack and pays \$29 in cash.
 - Eddie gets the tent and pays \$88 in cash.

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The Method of Sealed Bids

Example (2 Players, 4 Assets)

- Two children, Andy, Bob, are dividing four assets.
- The assets are four paintings

The Method of Sealed Bids

Example (The Bidding)

	Painting of Grandma	Painting of Grandpa	Painting of Great Grandma	Painting of Great Grandpa	Total Value	Fair Share
Andy	100	300	500	600		
Bob	250	200	300	350		

The Bids

The Method of Sealed Bids

Example (The Bidding)

	Painting of Grandma	Painting of Grandpa	Painting of Great Grandma	Painting of Great Grandpa	Total Value	Fair Share
Andy	100	300	500	600	1500	
Bob	250	200	300	350	1100	

The Bids

The Method of Sealed Bids

Example (The Bidding)

	Painting of Grandma	Painting of Grandpa	Painting of Great Grandma	Painting of Great Grandpa	Total Value	Fair Share
Andy	100	300	500	600	1500	750
Bob	250	200	300	350	1100	550

The Bids

The Method of Sealed Bids

Example (The Allocation of Assets)

- Andy gets
- Chuck gets

The Method of Sealed Bids

Example (The Allocation of Assets)

- Andy gets the paintings of Grandpa, Great-grandma, and Great-grandpa, worth \$1400.
- Chuck gets

The Method of Sealed Bids

Example (The Allocation of Assets)

- Andy gets the paintings of Grandpa, Great-grandma, and Great-grandpa, worth \$1400.
- Chuck gets the painting of Grandma, worth \$250.

The Method of Sealed Bids

Example (The First Settlement)

- Andy
- Bob

The Method of Sealed Bids

Example (The First Settlement)

- Andy **pays \$650** in cash.
- Bob

The Method of Sealed Bids

Example (The First Settlement)

- Andy **pays \$650** in cash.
- Bob **gets \$300** in cash.

The Method of Sealed Bids

Example (Dividing the Surplus)

- The surplus is

The Method of Sealed Bids

Example (Dividing the Surplus)

- The surplus is **\$350**.

The Method of Sealed Bids

Example (Dividing the Surplus)

- The surplus is **\$350**.
- So, each player gets

The Method of Sealed Bids

Example (Dividing the Surplus)

- The surplus is **\$350**.
- So, each player gets **\$175**.

The Method of Sealed Bids

Example (The Final Settlement)

- The final settlement is
 - Andy gets
 - Bob gets

The Method of Sealed Bids

Example (The Final Settlement)

- The final settlement is
 - Andy gets the paintings Grandpa, Great-grandma, and Great-grandpa and pays \$475 in cash.
 - Bob gets

The Method of Sealed Bids

Example (The Final Settlement)

- The final settlement is
 - Andy gets the paintings Grandpa, Great-grandma, and Great-grandpa and pays \$475 in cash.
 - Bob gets the painting of Grandma and gets \$475 in cash.

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Assignment

Assignment

- Chapter 3: Exercises 43, 44, 46, 47, 48.