

Hamilton's Method

Lecture 20
Sections 4.2

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1 Definitions

2 Hamilton's Method

3 Assignment

Outline

1 Definitions

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3 Assignment

Apportionment Problems

Definition (Apportionment Problem)

The classic **apportionment problem** involves a representative body where each **state** is given a certain number of **seats**, according to the state's **population**.

- Let N be the number of states.
- Let M be the number of seats.
- Let $p_1, p_2, p_3, \dots, p_N$ be the states' populations.
- Let $P = p_1 + p_2 + p_3 + \dots + p_N$, the total population.

Apportionment Problems

- More generally,
 - Let M be the number of things to be allocated (candies, seats).
 - Let N be the number of entities receiving the allocated things (students, states).
 - Let $p_1, p_2, p_3, \dots, p_N$ form the basis on which to allocate proportionally (points on test, population).
 - Let $P = p_1 + p_2 + p_3 + \dots + p_N$, the total, used to form the fractions (total points, total population).

Definitions

Definition (Standard Divisor)

The **standard divisor** (SD) is $\frac{P}{M}$. It represents the number of people that each seat represents.

$$SD = \frac{P}{M}.$$

Definition (Standard Quota)

The **standard quota** of a state is the exact fractional number of seats it should get for its “fair share.” It is computed as

$$q_i = \left(\frac{p_i}{P} \right) M = \frac{p_i}{SD}.$$

Definition (Lower and Upper Quotas)

The **lower quota** and the **upper quota** for a state are the two whole numbers nearest the standard quota for that state. If the standard quota happens to be a whole number, then the lower and upper quotas are the same.

Example

Example (Example)

- Suppose that three states A , B , and C , have populations 3 million, 6 million, and 7 million.
- Find the total population.

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- Suppose there are 50 seats to be apportioned.
- Find the standard divisor SD .

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- Find each state's standard quota.

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$$q_2 = 6/0.32 = 18.75$$

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- Find the lower and upper quotas for each state.

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- Find the total population. **16 million**
- Suppose there are 50 seats to be apportioned.
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- Find the lower and upper quotas for each state. **$(9, 10)$, $(18, 19)$, $(21, 22)$**

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Definition (Hamilton's Method)

- 1 Calculate the standard divisor SD .
- 2 Calculate each state's standard quota q_i .
- 3 Initially, give each state its lower quota.
- 4 Distribute the surplus to the states with the *largest fractional parts*.

Example

Example (Example)

- Apply Hamilton's method to the three states A , B , and C , with populations 3 million, 6 million, and 7 million and 50 seats to be apportioned.

Example – CA, TX, AK, WY

Example

- The populations of VA, NY, and OH are 8,001,024; 11,536,504; and 19,378,102 people, respectively.
- The total number of seats apportioned to those states is 55.
- Use Hamilton's method to apportion 55 seats.

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- Chapter 4: Exercises 11, 12, 13, 14, 19, 20.