The Huntington-Hill Method – Version 2

Lecture 24
Section 4.5

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Outline

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- Version 2 – Algorithmic
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  - The state with the largest such quotient gets one more seat, so add 1 to its quota q.
  - Repeat the previous 2 steps until all the seats have been apportioned.
  - Note that on each iteration only the q that was changed and its quotient need to be updated.

Outline

Example

Example (Example – Version 2)

- The populations of three states are $3, 7$ and 10 million people, respectively.
- The total number of seats apportioned to those states is 7.
- Use Version 2 to determine how many seats each state should get.

Example

Example (Example – Version 2)

State	Population (p)	Seats (q)	$D = \sqrt{q(q+1)}$	p/D
A	3	1		
B	7	1		
C	10	1		

Example

Example (Example – Version 2)

State	Population (p)	Seats (q)	$D = \sqrt{q(q+1)}$	p/D
A	3	1	$\sqrt{1 \cdot 2} = 1.414$	
B	7	1	$\sqrt{1 \cdot 2} = 1.414$	
C	10	1	$\sqrt{1 \cdot 2} = 1.414$	

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State	Population (p)	Seats (q)	$D = \sqrt{q(q+1)}$	p/D
A	3	1	$\sqrt{1 \cdot 2} = 1.414$	$\frac{3}{\sqrt{2}} = 2.121$
B	7	1	$\sqrt{1 \cdot 2} = 1.414$	
C	10	1	$\sqrt{1 \cdot 2} = 1.414$	

Example

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State	Population (p)	Seats (q)	$D = \sqrt{q(q+1)}$	p/D
A	3	1	$\sqrt{1 \cdot 2} = 1.414$	$\frac{3}{\sqrt{2}} = 2.121$
B	7	1	$\sqrt{1 \cdot 2} = 1.414$	$\frac{7}{\sqrt{2}} = 4.950$
C	10	1	$\sqrt{1 \cdot 2} = 1.414$	

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A	3	1	$\sqrt{1 \cdot 2} = 1.414$	$\frac{3}{\sqrt{2}} = 2.121$
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C	10	2	$\sqrt{1 \cdot 2} = 1.414$	$\frac{10}{\sqrt{2}} = 7.071$

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B	7	1	$\sqrt{1 \cdot 2} = 1.414$	$\frac{7}{\sqrt{2}} = 4.950$
C	10	2	$\sqrt{2 \cdot 3} = 2.449$	$\frac{10}{\sqrt{2}} = 7.071$

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State	Population (p)	Seats (q)	$D = \sqrt{q(q+1)}$	p/D
A	3	1	$\sqrt{1 \cdot 2} = 1.414$	$\frac{3}{\sqrt{2}} = 2.121$
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C	10	2	$\sqrt{2 \cdot 3} = 2.449$	$\frac{10}{\sqrt{6}} = 4.082$

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A	3	1	$\sqrt{1 \cdot 2} = 1.414$	$\frac{3}{\sqrt{2}} = 2.121$
B	7	2	$\sqrt{2 \cdot 3} = 2.449$	$\frac{7}{\sqrt{6}} = 2.858$
C	10	2	$\sqrt{2 \cdot 3} = 2.449$	$\frac{10}{\sqrt{6}} = 4.082$

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B	7	2	$\sqrt{2 \cdot 3} = 2.449$	$\frac{7}{\sqrt{6}} = 2.858$
C	10	3	$\sqrt{3 \cdot 4} = 3.464$	$\frac{10}{\sqrt{6}} = 4.082$

Example

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State	Population (p)	Seats (q)	$D = \sqrt{q(q+1)}$	p/D
A	3	1	$\sqrt{1 \cdot 2} = 1.414$	$\frac{3}{\sqrt{2}} = 2.121$
B	7	2	$\sqrt{2 \cdot 3} = 2.449$	$\frac{7}{\sqrt{6}} = 2.858$
C	10	3	$\sqrt{3 \cdot 4} = 3.464$	$\frac{10}{\sqrt{12}} = 2.886$

Example

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State	Population (p)	Seats (q)	$D = \sqrt{q(q+1)}$	p/D
A	3	1	$\sqrt{1 \cdot 2} = 1.414$	$\frac{3}{\sqrt{2}} = 2.121$
B	7	2	$\sqrt{2 \cdot 3} = 2.449$	$\frac{7}{\sqrt{6}} = 2.858$
C	10	3	$\sqrt{3 \cdot 4} = 3.464$	$\frac{10}{\sqrt{12}} = 2.886$

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A	3	1	$\sqrt{1 \cdot 2} = 1.414$	$\frac{3}{\sqrt{2}} = 2.121$
B	7	2	$\sqrt{2 \cdot 3} = 2.449$	$\frac{7}{\sqrt{6}} = 2.858$
C	10	4	$\sqrt{3 \cdot 4} = 3.464$	$\frac{10}{\sqrt{12}} = 2.886$

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State	Population (p)	Seats (q)	$D = \sqrt{q(q+1)}$	p/D
A	3	1	$\sqrt{1 \cdot 2} = 1.414$	$\frac{3}{\sqrt{2}} = 2.121$
B	7	2	$\sqrt{2 \cdot 3} = 2.449$	$\frac{7}{\sqrt{6}} = 2.858$
C	10	4	$\sqrt{4 \cdot 5} = 4.472$	$\frac{10}{\sqrt{12}} = 2.886$

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State	Population (p)	Seats (q)	$D = \sqrt{q(q+1)}$	p/D
A	3	1	$\sqrt{1 \cdot 2} = 1.414$	$\frac{3}{\sqrt{2}} = 2.121$
B	7	2	$\sqrt{2 \cdot 3} = 2.449$	$\frac{7}{\sqrt{6}} = 2.858$
C	10	4	$\sqrt{4 \cdot 5} = 4.472$	$\frac{10}{\sqrt{20}} = 2.236$

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State	Population (p)	Seats (q)	$D = \sqrt{q(q+1)}$	p/D
A	3	1	$\sqrt{1 \cdot 2} = 1.414$	$\frac{3}{\sqrt{2}} = 2.121$
B	7	2	$\sqrt{2 \cdot 3} = 2.449$	$\frac{7}{\sqrt{6}} = 2.858$
C	10	4	$\sqrt{4 \cdot 5} = 4.472$	$\frac{10}{\sqrt{20}} = 2.236$

Example

Example (Example – Version 2)

- The populations of WY, VT, ND, RI, NH, and NE are 564, 626, 673, 1053, 1316, and 1826 thousand people, respectively.
- The total number of seats apportioned to those states is 10.
- Use Version 2 to determine how many seats each state should get.

Outline

Which Version to Use?

- Suppose we had 3 states, with populations 2, 5, and 8 million, and 100 seats to apportion.
- Which method would be faster?

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- Suppose we had 3 states, with populations 2, 5, and 8 million, and 100 seats to apportion.
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- Why?

Which Version to Use?

- Suppose we had 8 states, with populations 1, 2, 4, 5, 8, 10, 13, and 14 million, and 9 seats to apportion.
- Which method would be faster?

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- Suppose we had 8 states, with populations 1, 2, 4, 5, 8, 10, 13, and 14 million, and 9 seats to apportion.
- Which method would be faster?
- Why?
- Work this example with $M = 12$.

Outline

Assignment

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- Chapter 4 Exercises 49, 50. Use Version 2 with $M = 10$.