

Apportionment Paradoxes

Lecture 25
Section 4.6

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Mon, Oct 29, 2018

- 1 Some History
- 2 The Quota Rule
- 3 Apportionment Paradoxes
 - The Alabama Paradox
 - The Population Paradox
 - The New-States Paradox
- 4 The Current Congress
- 5 Assignment

Outline

- 1 Some History
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History

- Jefferson's method was used in every apportionment from 1790 through 1830.
- In 1840, Congress adopted Webster's method.
- From 1850 through 1900, Hamilton's and Webster's methods were used. In each case, they produced the same result.
- From 1910 through 1930, Webster's method was used.
- The size of the House was steadily increased until 1929 when it was fixed at 435 seats.
- From 1940 to today, the Huntington-Hill method has been used.

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The Quota Rule

Definition (The Quota Rule)

The **quota rule** says that the number of representatives apportioned to each state should be either that state's lower quota or that state's upper quota (either L or U).

- As we have already seen, Hamilton's method is the only method that is *guaranteed* to satisfy the quota rule.

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- Is the quota rule fair?

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- Is the quota rule fair?
- Would it be unfair to violate the quota rule?

The Quota Rule

Example (Jefferson's Method and the Quota Rule)

- Consider CA (37254), AK (710), ND (673), VT (626), and WY (564).
- Those five states currently hold 57 seats altogether.
- Apportion 57 seats by the different methods.
- Apportion 100 seats by the different methods.

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The Alabama Paradox

- After the 1880 census, Congress had to decide how many House seats there would be, and then apportion them.
- If they created 299 House seats, then Alabama would get 8 seats.
- But if they created 300 House seats, then Alabama would get only 7 seats.

The Alabama Paradox

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- If they created 299 House seats, then Alabama would get 8 seats.
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- How can that be?

The Alabama Paradox

Definition (The Alabama Paradox)

The **Alabama paradox** occurs when a state is apportioned *fewer* seats when one new seat is *added*, even though none of the populations changed.

The Alabama Paradox

Example (Stolen from Wikipedia)

- Let states A , B , and C have populations of 2.1, 6.2, and 6.3 million.
- Compute the apportionment, under Hamilton's method, if there are 10 seats total.
- Add one seat for a total of 11 and reapportion.

The Alabama Paradox

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- Let states A , B , and C have populations of 2.1, 6.2, and 6.3 million.
- Compute the apportionment, under Hamilton's method, if there are 10 seats total.
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- Does the same thing happen under the other methods?

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The Population Paradox

- From 1890 to 1900, Virginia's population grew much faster than Maine's population.

State	1890 Pop	1900 Pop	Increase	% Incr
Virginia	1,655,980			
Maine	661,086			

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The Population Paradox

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- However, when the seats were reapportioned, Virginia lost a seat and Maine gained a seat.
- How can that be?

The Population Paradox

Definition (The Population Paradox)

The **population paradox** occurs when one state loses a seat and another state gains a seat, even though the first state's population increased *more* than the second state's population (either the absolute increase or the percentage increase).

The Population Paradox

Example (The Population Paradox)

- Let states A , B , and C have populations 530, 990, and 2240 thousand, respectively, with 24 seats to be apportioned.
- Calculate the number of seats apportioned, using Hamilton's method.
- Increase A 's population to 680 thousand, B 's population to 1250 thousand, and C 's population to 2570 thousand and recalculate the apportionment.

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The New-States Paradox

- In 1907, Oklahoma was admitted to the union.
- There were 386 seats in the House.
- Based on Oklahoma's population, it deserved to get 5 seats, so the total was raised to 391 seats.
- When the seats were reapportioned, Maine gained a seat and New York lost a seat.

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The New-States Paradox

Definition (The New-States Paradox)

The **new-states paradox** occurs when a new state is added and the number of seats is increased by the new state's fair share, yet the number of seats apportioned to the other states changes.

The New-States Paradox

Example (The New-States Paradox)

- Let states A and B have populations 52 and 134 million, respectively, with 16 seats to be apportioned.
- Calculate the number of seats apportioned, using Hamilton's method.
- Add a new state C with a population of 39 million and recalculate the apportionment.

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The Current Congress

The Current Congress

- Calculate the apportionment of the 115th Congress (the current Congress) using the Huntington-Hill method.
- Recalculate it, using the other four methods: Hamilton's, Jefferson's, Adams's, and Webster's.
- Are there any differences?

Comparisons with the Current Congress

Example (Comparisons with the Current Congress)

State	Ham	Jeff	Adams	Web	Hill
CA	53	55	50	53	53
DE	1	1	2	1	1
FL	27	28	26	27	27
GA	14	14	13	14	14
ID	2	2	3	2	2
IL	18	19	18	18	18
IA	4	4	5	4	4
LA	6	6	7	6	6
ME	2	1	2	2	2
MN	8	7	8	8	8
MO	8	8	9	8	8
MT	1	1	2	1	1
NE	3	2	3	3	3
NH	2	1	2	2	2

Comparisons with the Current Congress

Example (Comparisons with the Current Congress)

State	Ham	Jeff	Adams	Web	Hill
NJ	12	13	12	12	12
NY	27	28	26	27	27
NC	13	14	13	14	13
OH	16	17	16	16	16
OK	5	5	6	5	5
OR	5	5	6	5	5
RI	2	1	2	1	2
SC	7	6	7	7	7
SD	1	1	2	1	1
TX	36	37	34	36	36
VT	1	0	1	1	1
WA	10	10	9	10	10
WV	3	2	3	3	3
WY	1	0	1	1	1

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- Ch. 4: Exercises 51, 52, 55, 56, 58, 61, 62.