

# Introduction to Game Theory

## Lecture 34

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- 1 Introduction
- 2 Workers and Shirkers
- 3 The Prisoners' Dilemma
- 4 Assignment

# Outline

1 Introduction

2 Workers and Shirkers

3 The Prisoners' Dilemma

4 Assignment

# Introduction

- **Game Theory** was developed in the 1940s by John von Neumann and Oskar Morgenstern.
- It could more accurately be called **Strategy Theory**, or **The Theory of Strategic Interaction**.
- It has little to do with ordinary games. (chess, poker, rummy, Uno).

# Introduction

- **Game Theory** was developed in the 1940s by John von Neumann and Oskar Morgenstern.
- It could more accurately be called **Strategy Theory**, or **The Theory of Strategic Interaction**.
- It has little to do with ordinary games. (chess, poker, rummy, Uno).
- But it is still called Game Theory.

# The Players and Options

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- Each player selects one from among two or more **options** for his play.
- The outcome is determined by the combination of options that the two players selected.



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  - Each player knows the options available to both players.
  - Each player knows the **payoffs** resulting from their choices.
  - Neither player knows the other player's choice. They choose simultaneously.

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# Example

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- They each have the choice of whether to **work** with their partner or to **shirk** their responsibility.

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- Andy and Bob are to work together on a project.
- They each have the choice of whether to **work** with their partner or to **shirk** their responsibility.
- The combinations of choices are

Andy	Bob	Combination
work	work	(work, work)
work	shirk	(work, shirk)
shirk	work	(shirk, work)
shirk	shirk	(shirk, shirk)

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- It depends on their personalities.
  - Is Andy a “worker” or is he a “shirker?”
  - Is Bob a “worker” or is he a “shirker?”

# Andy and Bob, the two Shirkers

		Bob	
		Work	Shirk
Andy	Work	$(w, w)$	$(w, s)$
	Shirk	$(s, w)$	$(s, s)$

- Suppose that Andy and Bob are both shirkers.
- What happens?

# Andy and Bob, the two Shirkers

		Bob	
		Work	Shirk
Andy	Work	$(w, w)$	$(w, s)$
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- Suppose that Andy and Bob are both workers.
- What happens?

# Andy and Bob, the two Shirkers

		Bob	
		Work	Shirk
Andy	Work	$(w, w)$	$(w, s)$
	Shirk	$(s, w)$	$(s, s)$

- Suppose that Andy and Bob both understand that the project must get done, but they would each prefer to shirk.
- What happens?

# Preference Rankings

- For each player, we may assign a preference ranking to each outcome (1 = best, 4 = worst).
- Assume Andy is a worker and Bob is a shirker.

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Andy

Outcome	Rank
$(w, w)$	1
$(w, s)$	2
$(s, w)$	3
$(s, s)$	4

# Preference Rankings

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- Assume Andy is a worker and Bob is a shirker.

Andy	
Outcome	Rank
$(w, w)$	1
$(w, s)$	2
$(s, w)$	3
$(s, s)$	4

Bob	
Outcome	Rank
$(w, w)$	3
$(w, s)$	1
$(s, w)$	4
$(s, s)$	2



# Andy and Bob, the two Shirkers

		Bob	
		Work	Shirk
Andy	Work	(1, 3)	(2, 1)
	Shirk	(3, 4)	(4, 2)

- Andy is a worker and Bob is a shirker.
- $(x, y)$  shows Andy's ranking  $x$  and Bob's ranking  $y$ .
- Andy prefers **smaller**  $x$  values.
- Bob prefers **smaller**  $y$  values.
- What happens?

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- Andy is told that if he cooperates and Bob does not, then Andy will be set free while Bob will serve 2 years.

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- Otherwise, he will receive a harsher sentence.



# The Prisoners' Dilemma

- Now Andy and Bob finished college and went into the “banking business.”
- They are charged with bank robbery.
- However, the prosecutor does not have a strong case against them.
- So he separates them and gives them each a chance to plea bargain.
- Andy is told that if he cooperates and Bob does not, then Andy will be set free while Bob will serve 2 years.
- Otherwise, he will receive a harsher sentence.
- Yet if neither one cooperates, they might go free.

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# The Prisoners' Dilemma

- Andy and Bob each have two options: To cooperate or not cooperate.
- How would they rank the outcomes?
  - If they both cooperate, they both get moderate sentences.
  - If neither cooperates, they get lighter sentences.
  - If one cooperates and the other doesn't, then the one who cooperates goes free while the other one gets a harsher sentence.

# Andy and Bob, the two Prisoners

		Bob	
		Cooperate	Not Cooperate
Andy	Cooperate	$(c, c)$	$(c, n)$
	Not Cooperate	$(n, c)$	$(n, n)$

- What will Andy and Bob decide to do?

# Preference Rankings

- For each player, assign a preference ranking to each outcome (1 = best, 4 = worst).

Andy

Outcome	Sentence	Rank
$(c, c)$	Moderate	
$(c, n)$		
$(n, c)$		
$(n, n)$		

Bob

Outcome	Sentence	Rank
$(c, c)$	Moderate	
$(c, n)$		
$(n, c)$		
$(n, n)$		

# Preference Rankings

- For each player, assign a preference ranking to each outcome (1 = best, 4 = worst).

Andy		
Outcome	Sentence	Rank
$(c, c)$	Moderate	
$(c, n)$	None	
$(n, c)$		
$(n, n)$		

Bob		
Outcome	Sentence	Rank
$(c, c)$	Moderate	
$(c, n)$	Harsh	
$(n, c)$		
$(n, n)$		



# Preference Rankings

- For each player, assign a preference ranking to each outcome (1 = best, 4 = worst).

Andy		
Outcome	Sentence	Rank
$(c, c)$	Moderate	
$(c, n)$	None	
$(n, c)$	Harsh	
$(n, n)$		

Bob		
Outcome	Sentence	Rank
$(c, c)$	Moderate	
$(c, n)$	Harsh	
$(n, c)$	None	
$(n, n)$		

# Preference Rankings

- For each player, assign a preference ranking to each outcome (1 = best, 4 = worst).

Andy

Outcome	Sentence	Rank
$(c, c)$	Moderate	
$(c, n)$	None	
$(n, c)$	Harsh	
$(n, n)$	Light	

Bob

Outcome	Sentence	Rank
$(c, c)$	Moderate	
$(c, n)$	Harsh	
$(n, c)$	None	
$(n, n)$	Light	

# Preference Rankings

- For each player, assign a preference ranking to each outcome (1 = best, 4 = worst).

Andy		
Outcome	Sentence	Rank
$(c, c)$	Moderate	3
$(c, n)$	None	1
$(n, c)$	Harsh	4
$(n, n)$	Light	2

Bob		
Outcome	Sentence	Rank
$(c, c)$	Moderate	3
$(c, n)$	Harsh	4
$(n, c)$	None	1
$(n, n)$	Light	2

# The Prisoners' Dilemma

		Bob	
		Cooperate	Not Cooperate
Andy	Cooperate	(3, 3)	(1, 4)
	Not Cooperate	(4, 1)	(2, 2)

What will Andy and Bob do?

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# Assignment

## Assignment

- Work the problems on Handout #1.