

The Nash Equilibrium

Lecture 38

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Wed, Dec 5, 2018

1 Nash Equilibrium

2 John Nash

3 Examples

4 Assignment

Outline

- 1 Nash Equilibrium
- 2 John Nash
- 3 Examples
- 4 Assignment

Nash Equilibrium

Definition (Nash Equilibrium)

A **Nash equilibrium** is a pure or mixed strategy with the property that if the player deviates from it, his opponent can take advantage of that deviation. That is, the opponent, by changing his own strategy, can lower the player's average payoff.

Nash's Equilibrium Theorem

Theorem (Nash's Equilibrium Theorem)

Every game has at least one Nash equilibrium.

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Every game has at least one Nash equilibrium.

- A game may have more than one Nash equilibrium, but it must have at least one.
- If there is more than one Nash equilibrium, then one of them may be more favorable to the players than the other one.

Nash's Equilibrium Theorem

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Nash's Equilibrium Theorem

- The Nash Equilibrium Theorem implies that in any strategic situation that can be described by a payoff matrix, there exist mutually beneficial strategies for the players, even though the players are not cooperating.
- Because the situation is non-cooperative, a Nash equilibrium may not be the very best for the players, but it is a stable point.
- Recall the Prisoners' Dilemma, the Travelers' Dilemma, and the Diners' Dilemma.

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John Nash

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- In 1959, he was diagnosed with paranoid schizophrenia.

John Nash

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- He died in 2015 in a car wreck on the New Jersey Turnpike.
- He proved and published his theorem about equilibria in 1950 as part of his doctoral dissertation at Princeton University.
- In 1959, he was diagnosed with paranoid schizophrenia.
- In spite of that, he went on to have a stellar career, developing many theories in high mathematics and receiving a number of national awards.

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Pure Nash Equilibria

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		Bob	
		4♣	2♦
Andy	2♠	(2, 2)	(0, 0)
	7♥	(0, 0)	(1, 1)

- Andy and Bob each play one of their cards.
- If both cards are black, they both win \$2.
- If both cards are red, they both win \$1.
- If one card is black and the other is red, they win nothing.

Pure Nash Equilibria

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		Bob	
		4♣	2♦
Andy	2♠	(2, 2)	(0, 0)
	7♥	(0, 0)	(1, 1)

- There are two pure Nash equilibria:
 - Both players play black.
 - Both players play red.
- If either of them (alone) deviates from their pure strategy, they suffer loss.

Mixed Nash Equilibrium

Mixed Nash Equilibrium

		Bob	
		4♣	2♦
Andy	2♠	2	-2
	7♥	-2	2

- Andy and Bob each play one of their cards.
- If the cards are the same color, Andy wins \$2.
- If the cards are different colors, Bob wins \$2.

Mixed Nash Equilibrium

Mixed Nash Equilibrium

		Bob	
		4♣	2♦
Andy	2♠	2	-2
	7♥	-2	2

- The Nash equilibrium is that they both play their red and black cards, each 50% of the time and at random.
- For example, if Andy decides to play black more than 50% of the time, then Bob (once he realizes that) will play red more than 50% of the time (maybe 100% of the time).

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- No new assignment.