

# Installment Loans

## Lecture 6 Section 10.4

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1 Installment Loans

2 Examples

3 Assignment

# Outline

1 Installment Loans

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# Installment Loans

## Definition (Installment Loan)

With an **installment loan**, the borrower pays back the loan through regular, equal payments (usually monthly). For each payment,

- The interest rate is applied to the outstanding balance at the end of the payment period.
  - The interest is added to the outstanding balance.
  - And the payment is subtracted from the outstanding balance, producing the new balance to be carried over to the next payment period.
- 
- In order to pay the loan off, the payments must be greater than the interest earned between payments.
  - The greater the difference, the faster the loan is paid off.

# Example

## Example (Five Payments)

- A person borrows \$10,000.00 at 10% interest APR to be paid back in 5 annual payments.
- The annual payment turns out to be \$2,637.97.
- Trace the schedule of payments over the five years as interest is applied and the payments are made.

# Example

## Example (Five Payments)

| Year | Starting Balance | Interest | Total     | Payment  | Ending Balance |
|------|------------------|----------|-----------|----------|----------------|
| 1    | 10,000.00        | 1,000.00 | 11,000.00 | 2,637.97 | 8,362.03       |
|      |                  |          |           |          |                |
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| 3    | 6,560.26         | 656.03   | 7,216.29  | 2,637.97 | 4,578.32       |
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| 4    | 4,578.32         | 457.83   | 5,036.15  | 2,637.97 | 2,398.18       |
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| 4    | 4,578.32         | 457.83   | 5,036.15  | 2,637.97 | 2,398.18       |
| 5    | 2,398.18         | 239.82   | 2,638.00  | 2,637.97 | 0.03           |

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  - **You** write monthly checks to **the bank** (your payments), paying down that amount, plus interest.
- Annuity
  - **The bank** has “borrowed” a large sum of money from **you** (your savings account).
  - **The bank** writes monthly checks to **you** (your withdrawals), paying down that amount, plus interest.

# Installment Loans

- The formula for the payment  $M$  for an installment loan is exactly the same as the annuity formula for drawing down:

$$M = P \left( \frac{r/k}{1 - \left(1 + \frac{r}{k}\right)^{-kt}} \right),$$

where  $P$  is the principal (amount borrowed),  $r$  is the annual interest rate,  $k$  is the number of payments per year, and  $t$  is the number of years.

- Payments are usually monthly ( $k = 12$ ), but they do not need to be.

# Example

## Example (Three-year loan)

In our example,  $k = 1$ , so the payment is

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$$\begin{aligned}M &= \frac{Pr}{1 - (1 + r)^{-t}} \\ &= \frac{(10000)(.10)}{1 - (1.10)^{-5}} \\ &= \$2,637.97.\end{aligned}$$

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# Example

## Example (Financing a Car)

- You would like to purchase a new car priced at \$25,000.
- You can put \$5,000 down and finance the rest at 6% for 60 months.
- Find
  - The monthly payment.
  - The total amount paid.
  - The total amount of interest paid.

# Example

## Example (Home Mortgage)

- You would like to purchase a home for \$300,000.
- You can put \$60,000 down and finance the rest at 4.334% for 30 years, at 3.772% for 15 years, or at 3.838% for 10 years. (From Consumer Direct)
- Find
  - The monthly payment for each choice.
  - The total amount paid for each choice.
  - The total amount of interest paid for each choice.



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- You can put \$60,000 down and finance the rest at 4.334% for 30 years, at 3.772% for 15 years, or at 3.838% for 10 years. (From Consumer Direct)
- Find
  - The monthly payment for each choice.
  - The total amount paid for each choice.
  - The total amount of interest paid for each choice.
- Which is best? 30 years at 4.334%? 15 years at 3.772%? or 10 years at 3.838%?

# Example

## Example (Home Mortgage)

- Benchmark Community Bank [calculator](#).

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# Assignment

## Assignment

- Chapter 10: 57, 58, 59, 61, 62, 63.