

# Apportionment Paradoxes

Lecture 18  
Section 4.6

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- 1 The Quota Rule
- 2 Some History
- 3 Apportionment Paradoxes
  - The Alabama Paradox
  - The Population Paradox
  - The New-States Paradox
- 4 The Current Congress
- 5 Assignment

# Outline

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# The Quota Rule

## Definition (The Quota Rule)

The **quota rule** says that the number of representatives apportioned to each state should be at least that state's lower quota, but not more than that state's upper quota.

- As we have already seen, Hamilton's method is the only method that is *guaranteed* to satisfy the quota rule.
- Is the quota rule fair?

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# History

- Jefferson's method was used in every apportionment from 1790 through 1830.
- In 1840, Congress adopted Webster's method.
- From 1850 through 1900, Hamilton's and Webster's methods were used because they produced the same result.
- In 1929, the size of the House was fixed at 435 seats.
- From 1910 through 1930, Webster's method was used.
- From 1940 to today, the Huntington-Hill method has been used.

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- 4 The Current Congress
- 5 Assignment

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# The Alabama Paradox

- After the 1880 census, Congress had to decide how many House seats there would be, and then apportion them.
- If they created 299 House seats, then Alabama would get 8 seats.
- But if they created 300 House seats, then Alabama would get only 7 seats.

# The Alabama Paradox

## Definition (The Alabama Paradox)

The **Alabama paradox** occurs when a state is apportioned *fewer* seats when one new seat is *added*, even though none of the populations changed.

## Example (Stolen from Wikipedia)

- Let states  $A$ ,  $B$ , and  $C$  have populations 6, 6, and 2, in millions.
- Compute the apportionment, under Hamilton's method, if there are 10 seats total.
- Recompute the apportionment, under Hamilton's method, if there are 11 seats total.

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# The Population Paradox

- From 1890 to 1900, Virginia's population grew much faster than Maine's population.
- However, when the seats were reapportioned, Virginia lost a seat and Maine gained a seat.

# The Population Paradox

## Definition (The Population Paradox)

The **population paradox** occurs when one state loses a seat and another state gains a seat, even though the first state's population increased more than the second state's population.

## Example (The Population Paradox)

- Let states  $A$ ,  $B$ , and  $C$  have populations 13, 12, and 112 million, respectively, with 25 seats to be apportioned.
- Calculate the number of seats apportioned, using Hamilton's method.
- Add 1 million to  $A$ 's population and 2 million to  $C$ 's population and recalculate the apportionment.

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# The New-States Paradox

- In 1907, Oklahoma was admitted to the union.
- There were 386 seats in the House.
- Based on Oklahoma's population, it deserved to get 5 seats, so the total was raised to 391 seats.
- When the seats were reapportioned, Maine gained a seat and New York lost a seat.

# The New-States Paradox

## Definition (The New-States Paradox)

The **new-states paradox** occurs when a new state is added and the number of seats is increased by the new state's fair share, yet the number of seats apportioned to the other states changes.

## Example (The New-States Paradox)

- Let states  $A$  and  $B$  have populations 52 and 134 million, respectively, with 16 seats to be apportioned.
- Calculate the number of seats apportioned, using Hamilton's method.
- Add a new state  $C$  with a population of 39 million and recalculate the apportionment.



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# The Current Congress

## The Current Congress

- Calculate the apportionment of the 114th Congress using the Huntington-Hill method.
- Recalculate it, using the other four methods: Hamilton's, Jefferson's, Adams's, and Webster's.
- What are the differences?

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# Assignment

## Assignment

- Ch. 4: Exercises 51, 55, 56, 58, 61, 62.