The Critical-Path Algorithm

Lecture 34
Sections 8.3 - 8.4

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1. Critical Paths
2. The Backflow Algorithm
3. The Critical-Path Algorithm
4. Example
5. Assignment

Outline

1. Critical Paths
2. The Backflow Algorithm
3. The Critical-Path Algorithm
4. Example
5. Assignment

Definitions

Definition (Critical Path for a Vertex)

The critical path for a vertex is the path from that vertex to END with the longest processing time.

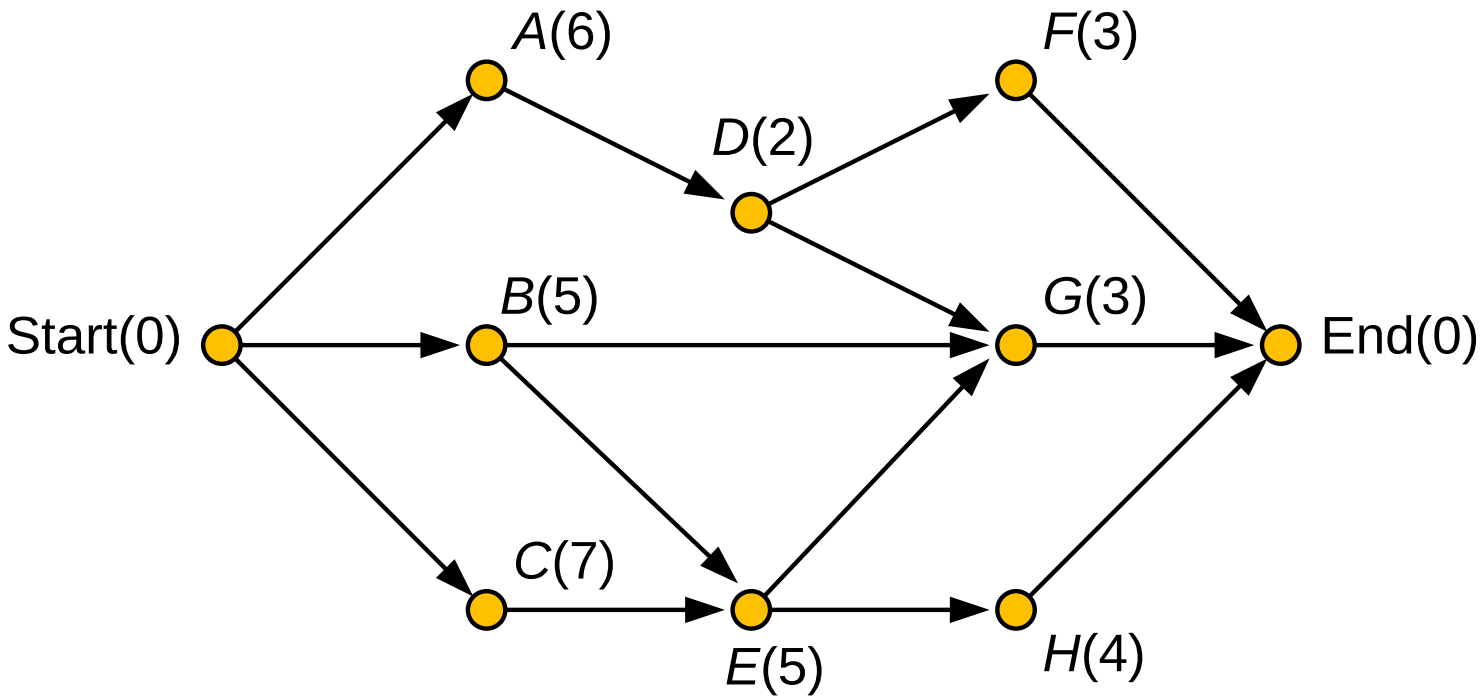
Definition (Critical Path for a Project)

The critical path for a project is the critical path from START to END.

Definition (Critical Time)

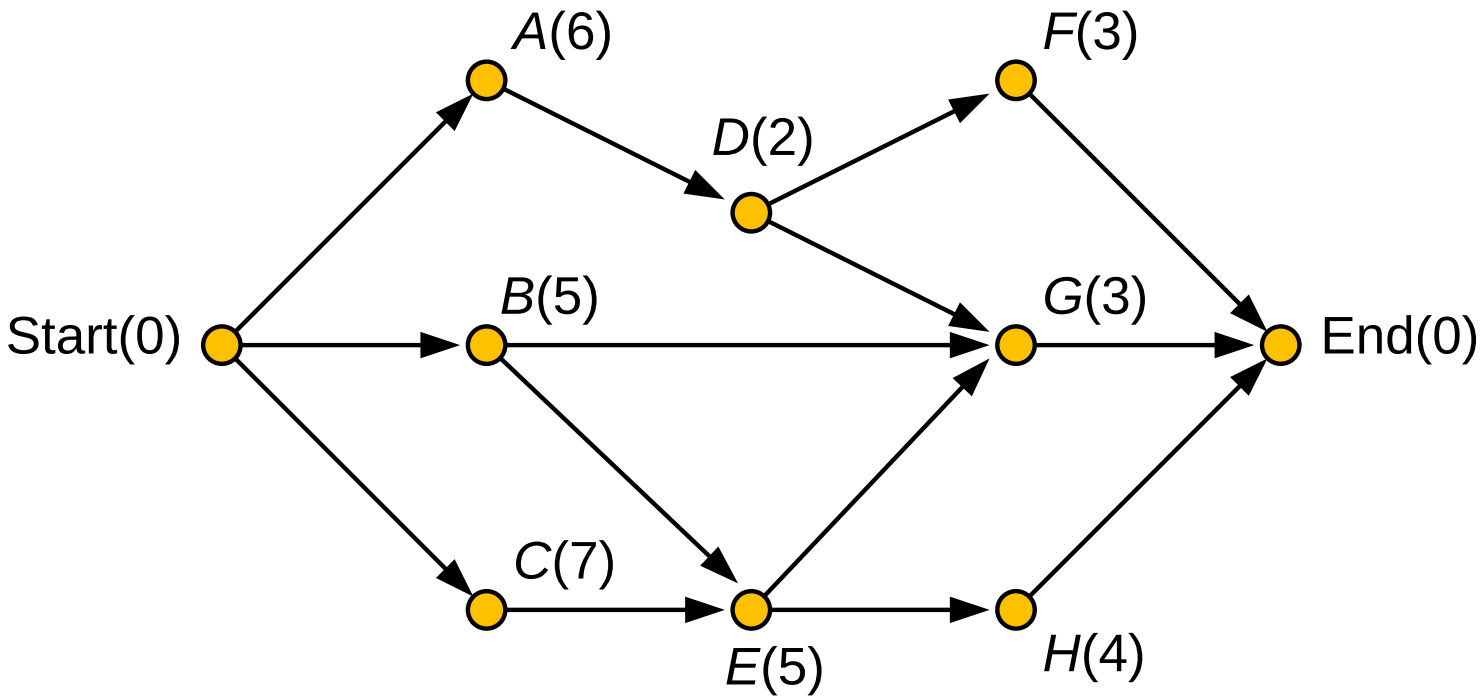
The critical time for a vertex or project is the processing time of its critical path.

Example



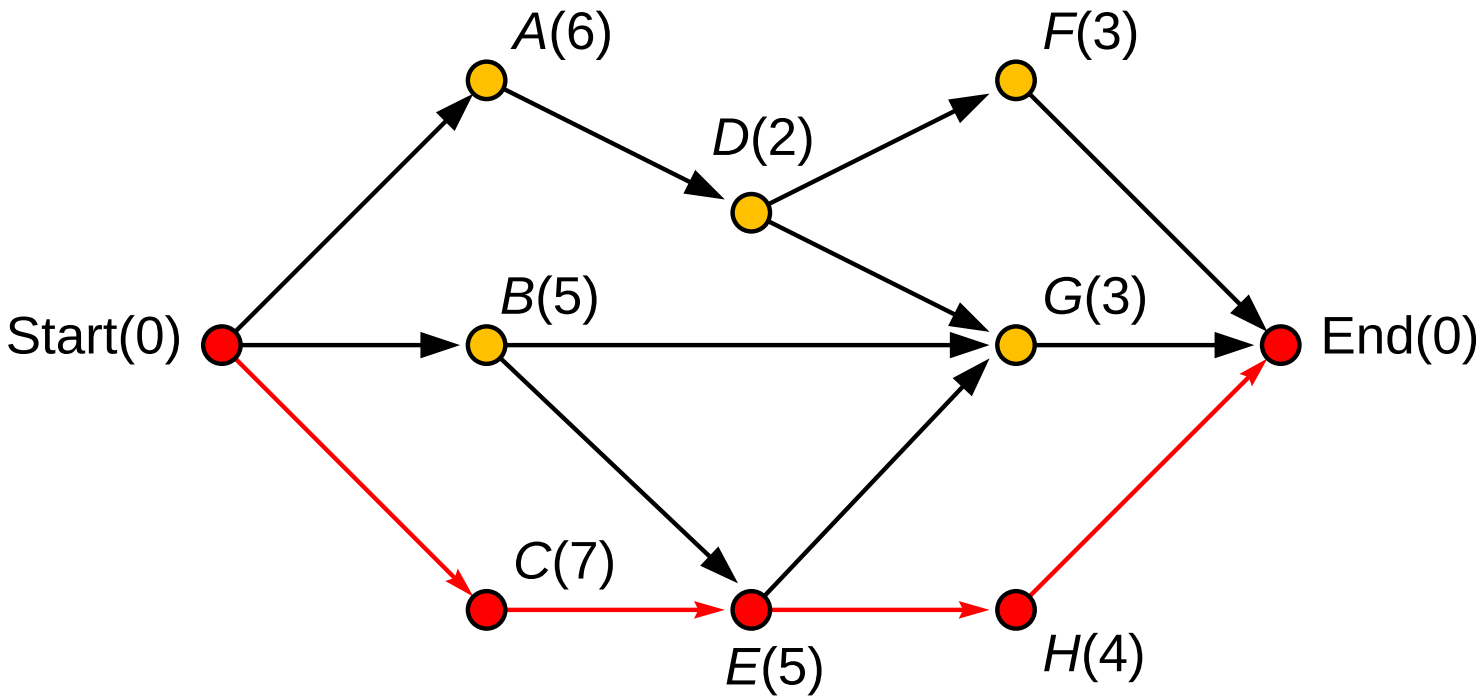
- The critical time of a project is the shortest possible time required to complete the project.

Example



- The critical time of a project is the shortest possible time required to complete the project.
- It is also the longest path (in terms of time) from START to END.

Example



- The critical time of a project is the shortest possible time required to complete the project.
- It is also the longest path (in terms of time) from START to END.

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The Backflow Algorithm

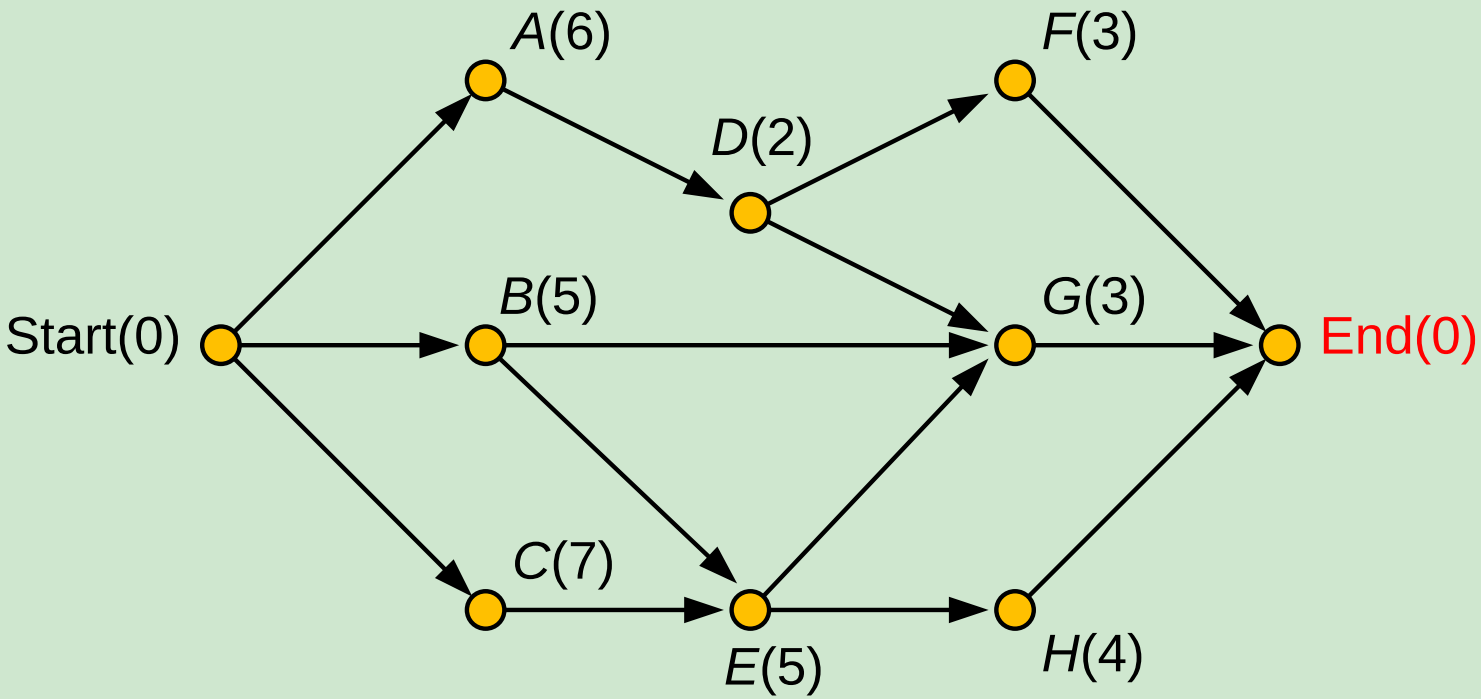
Definition (The Backflow Algorithm)

The backflow algorithm finds the critical path by the following method.

1. Beginning with END and working back to START, find the critical time for each vertex.
   - The critical time for a vertex is the processing time for that vertex plus the largest critical time of the vertices incident from that vertex.
2. The critical path for the project is the path from START to END whose edges connect each vertex to its successor with the greatest critical time.

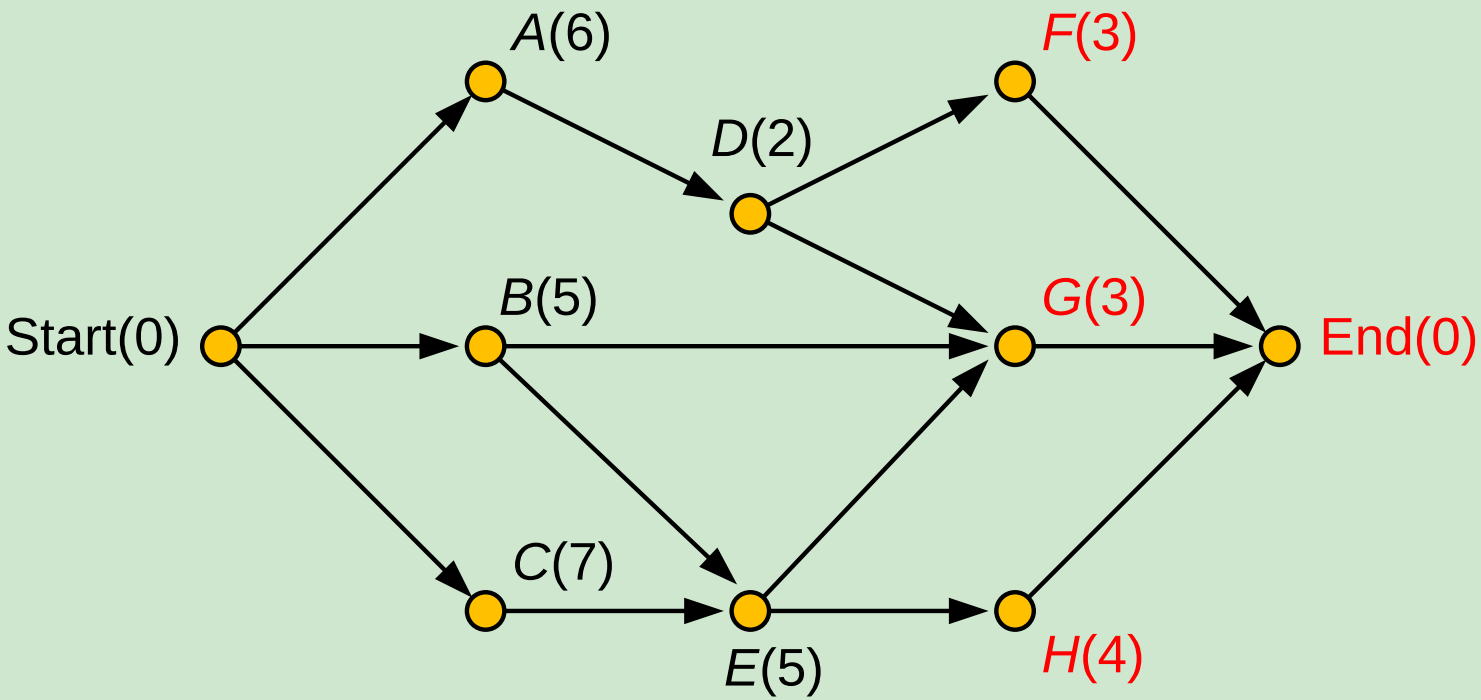
The Backflow Algorithm

Example (The Backflow Algorithm)



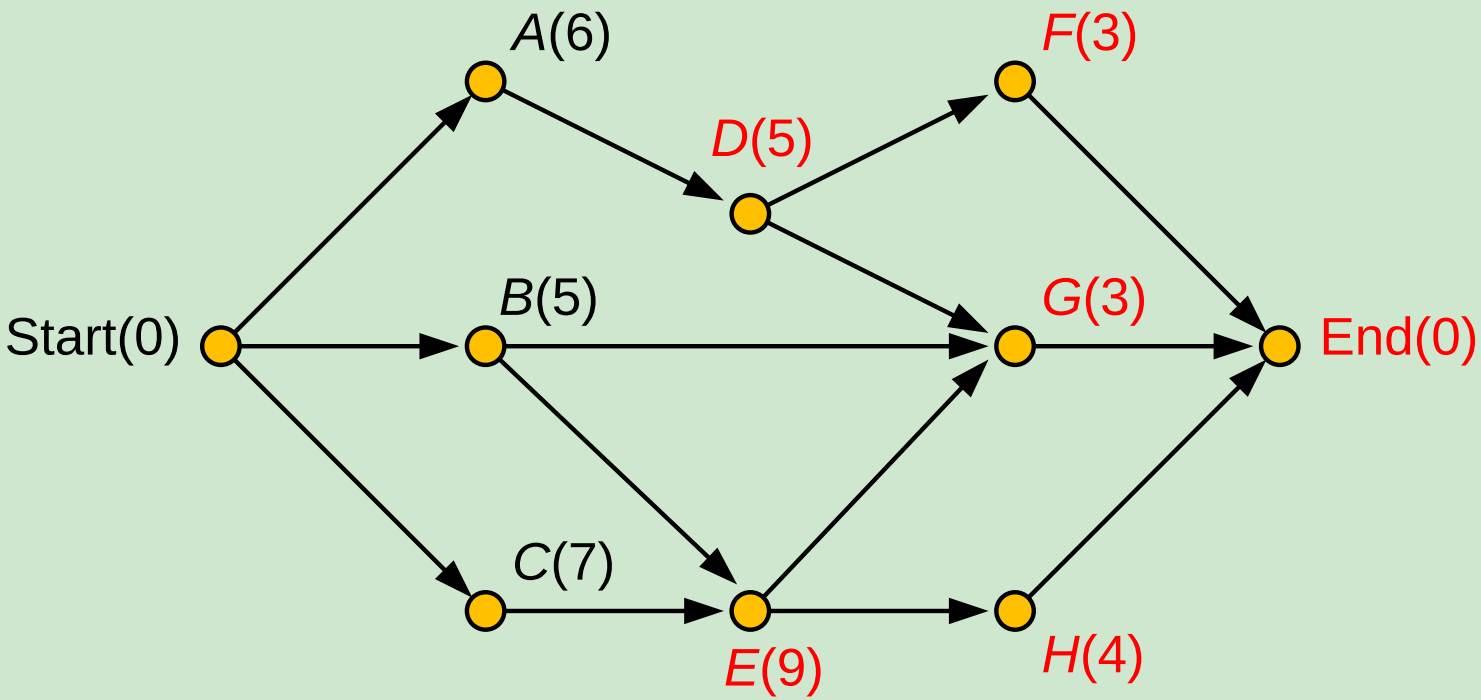
The Backflow Algorithm

Example (The Backflow Algorithm)



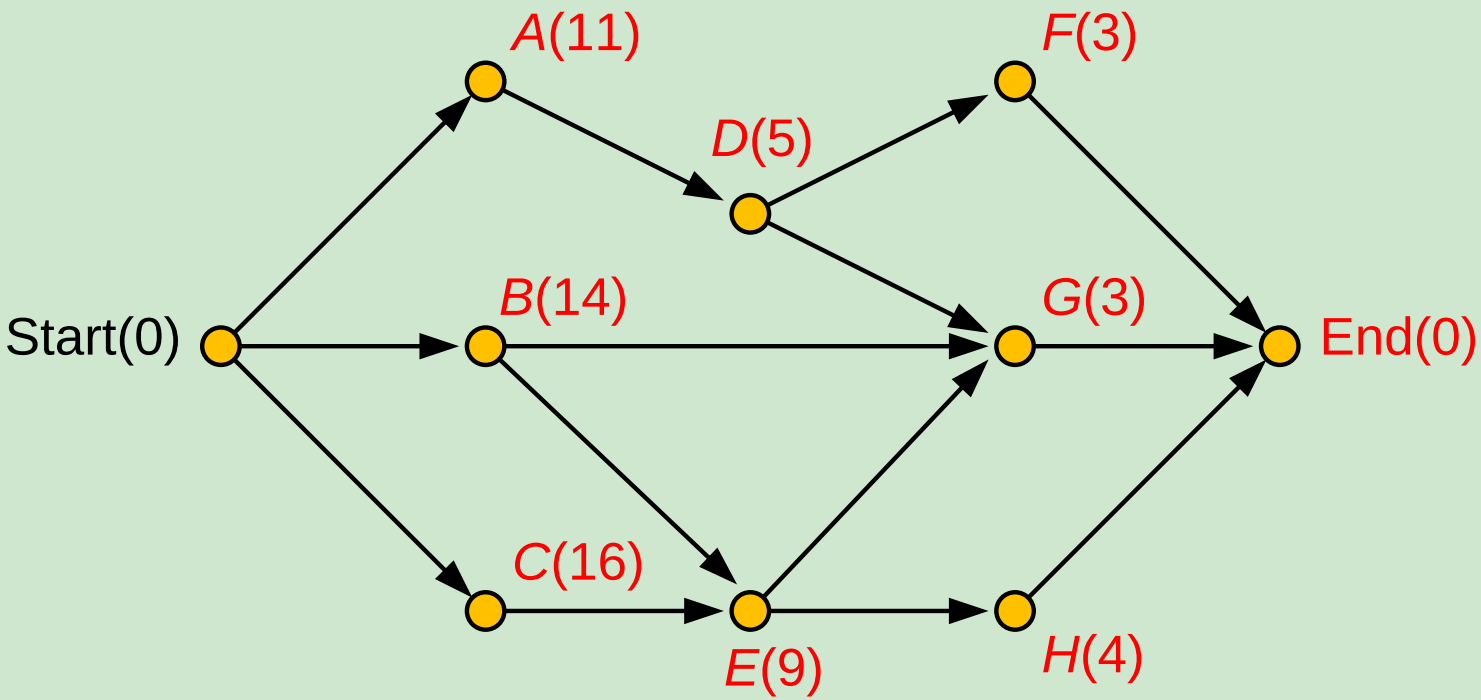
The Backflow Algorithm

Example (The Backflow Algorithm)



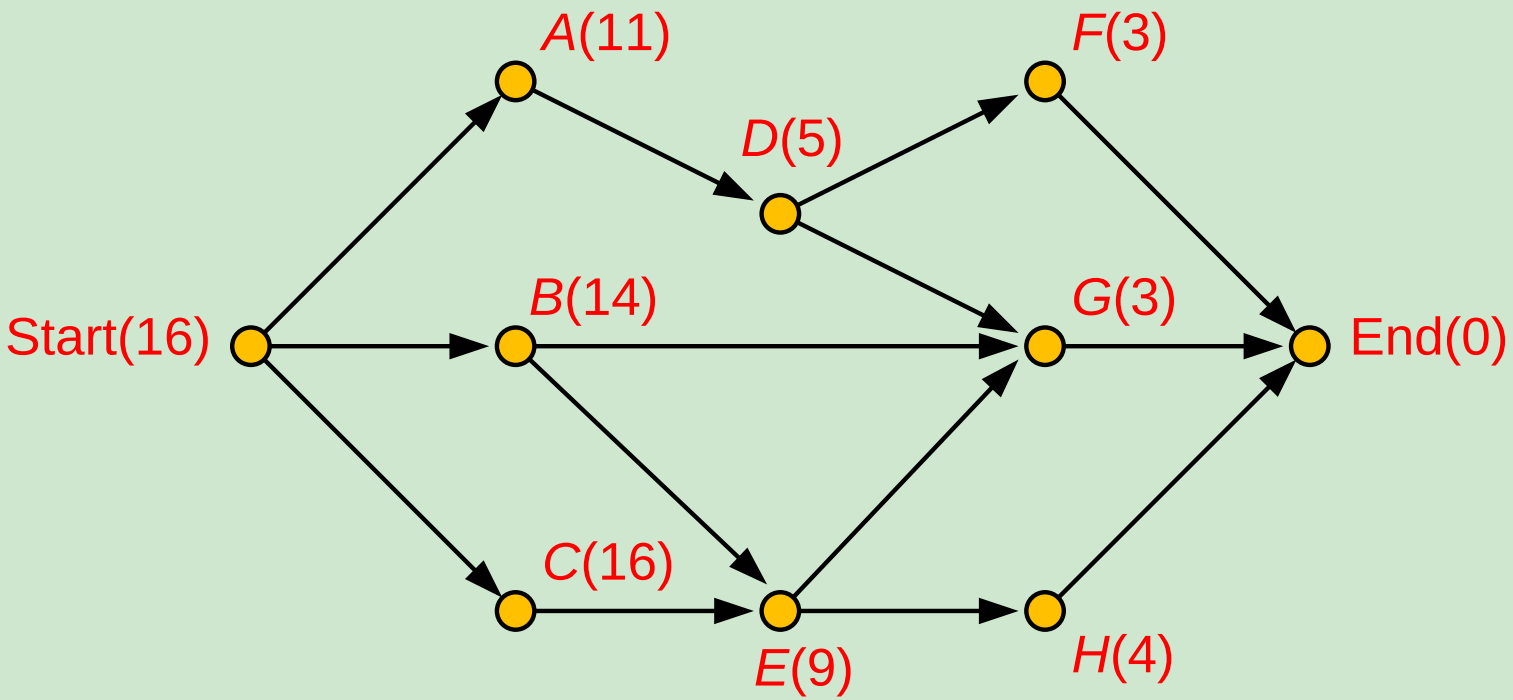
The Backflow Algorithm

Example (The Backflow Algorithm)



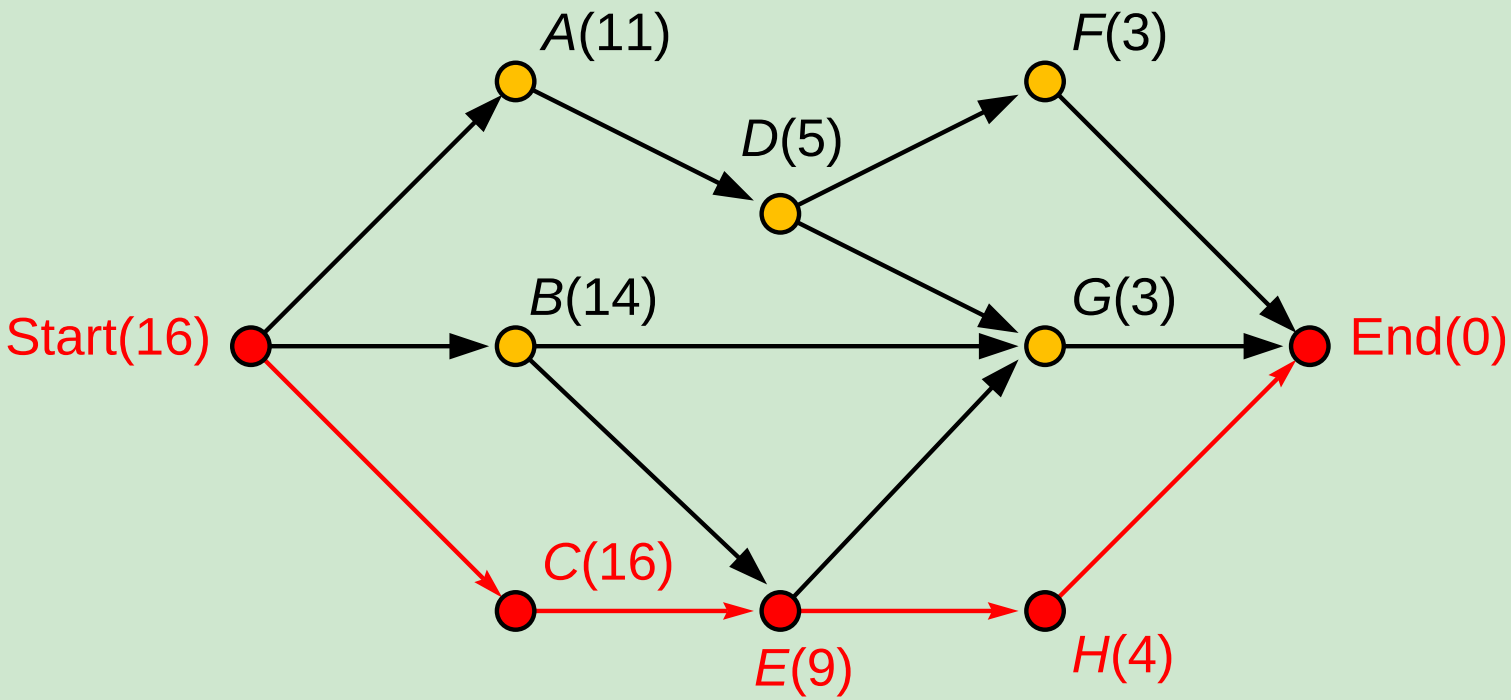
The Backflow Algorithm

Example (The Backflow Algorithm)



The Backflow Algorithm

Example (The Backflow Algorithm)



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The Critical-Path Algorithm

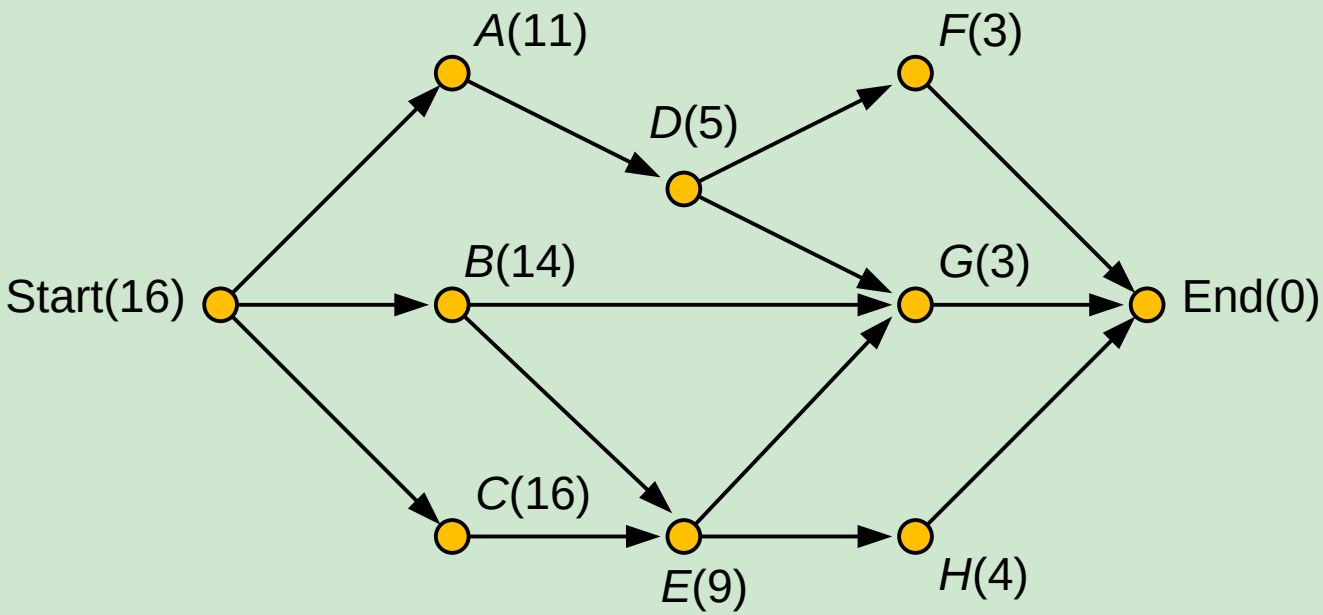
Definition (The Critical-Path Algorithm)

The critical-path algorithm creates a schedule by the following method.

1. Use the backflow algorithm to find the critical time of every task in the project.
2. Create a priority list with the tasks listed in order of decreasing *critical* time.
3. Use the priority list to create a schedule.

Example

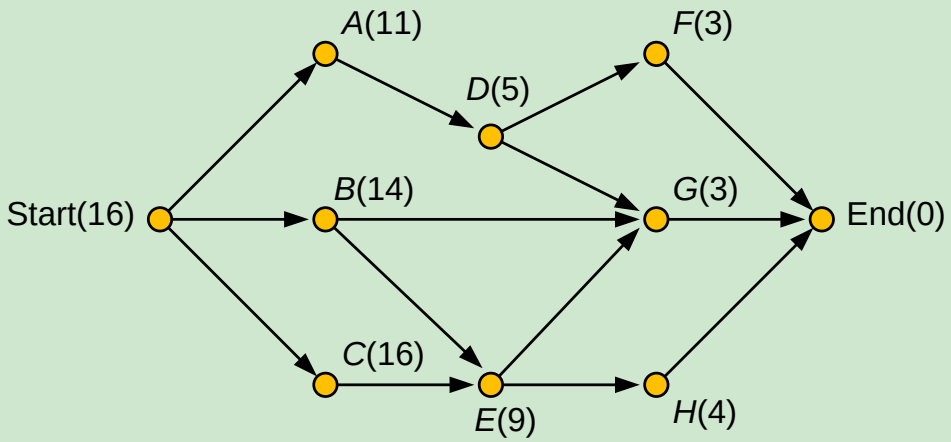
Example (The Critical-Path Algorithm)



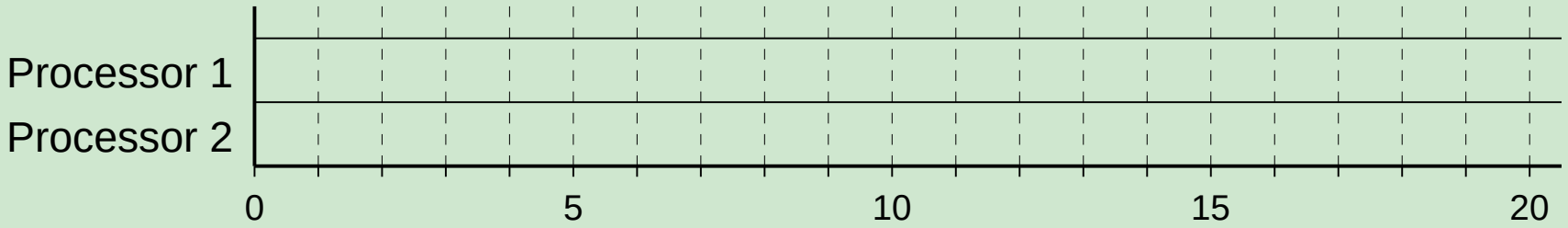
- The priority list is C, B, A, E, D, H, F, G.

Example

Example (The Critical-Path Algorithm)



Priority list: C, B, A, E, D, H, F, G



Example

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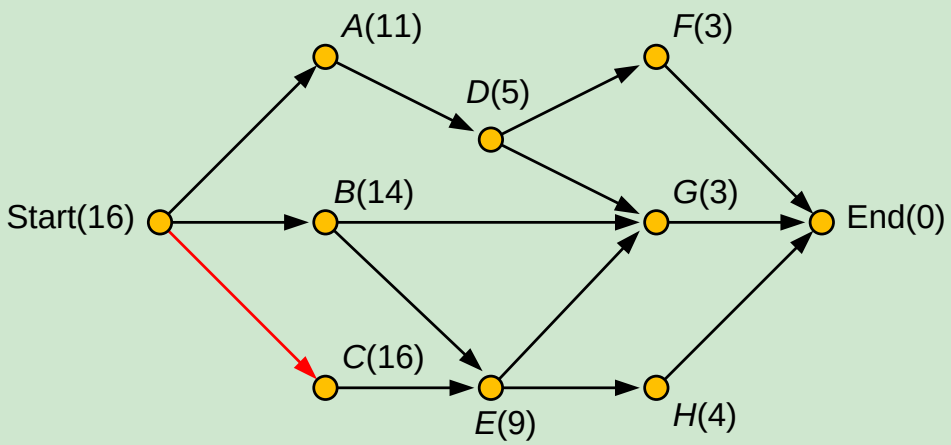
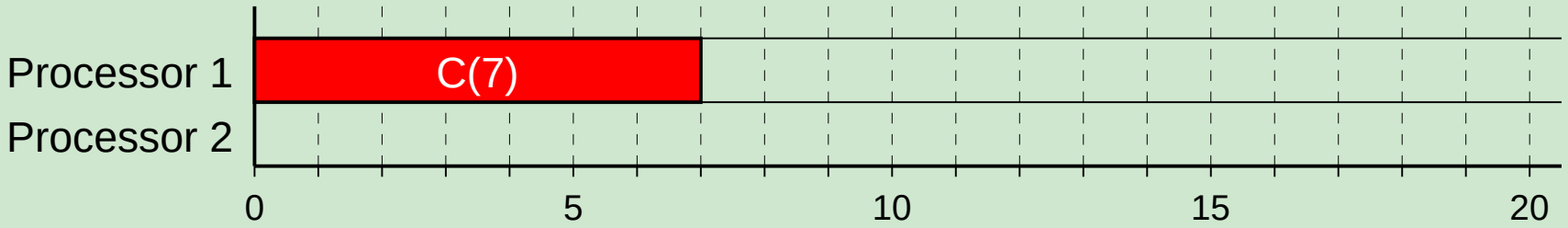


Priority list: C, B, A, E, D, H, F, G



Example

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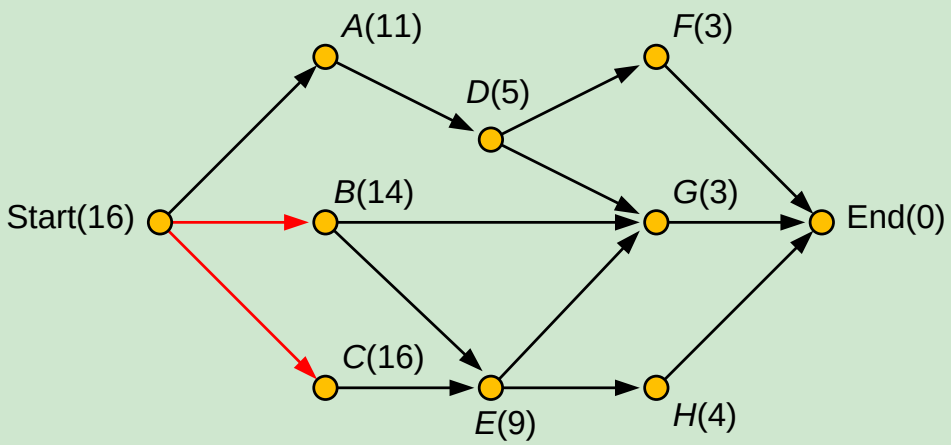
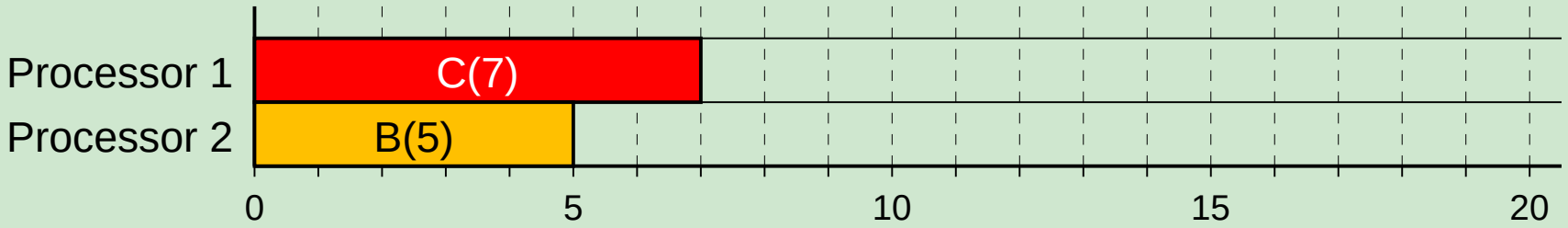


Priority list: *C*, *B*, *A*, *E*, *D*, *H*, *F*, *G*



Example

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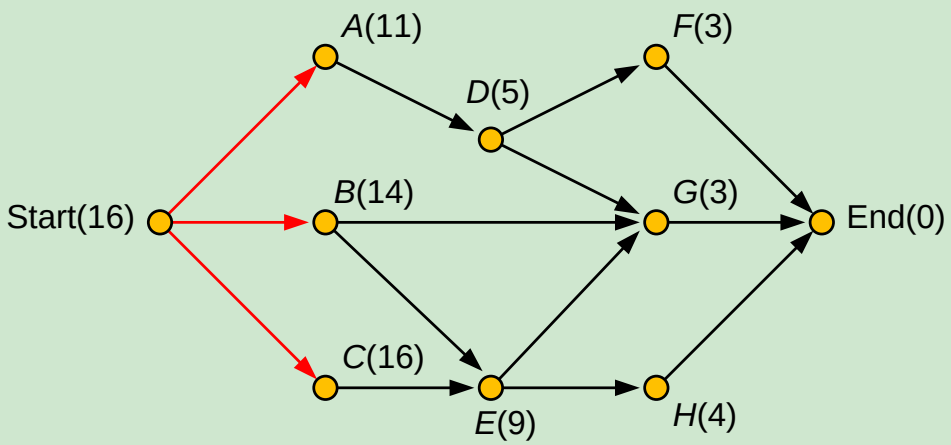

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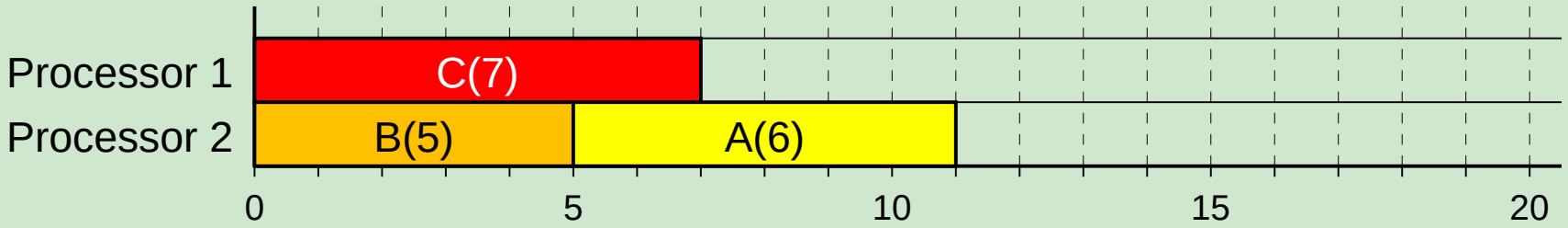

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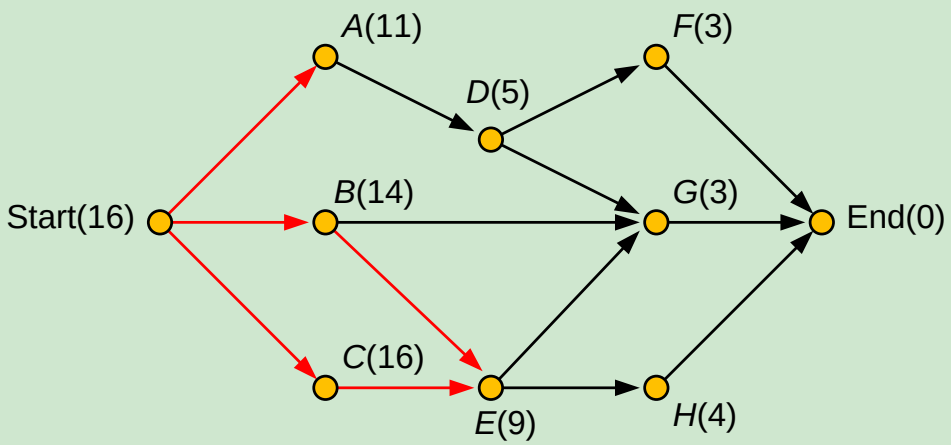

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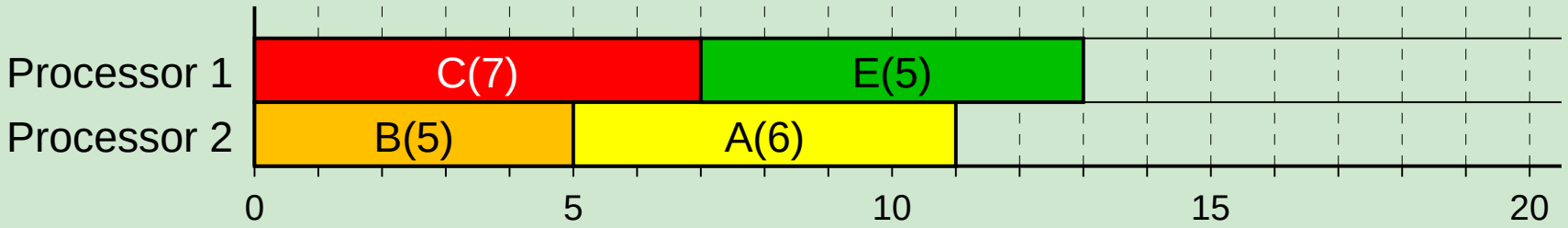

Example

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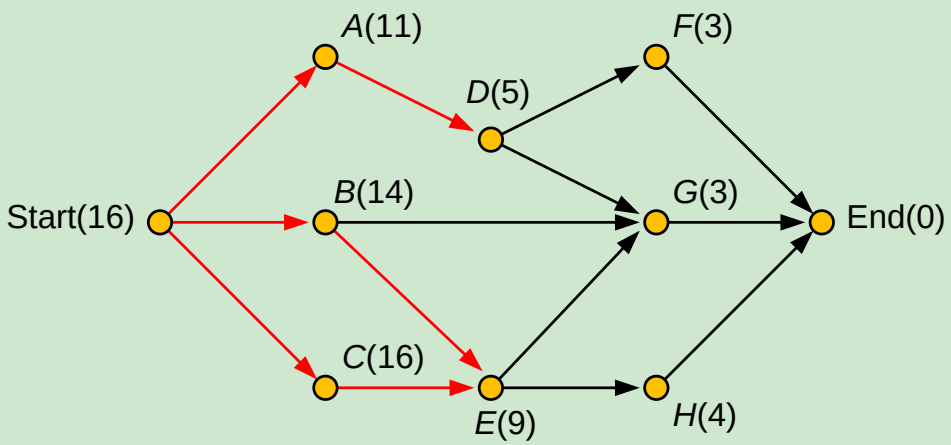

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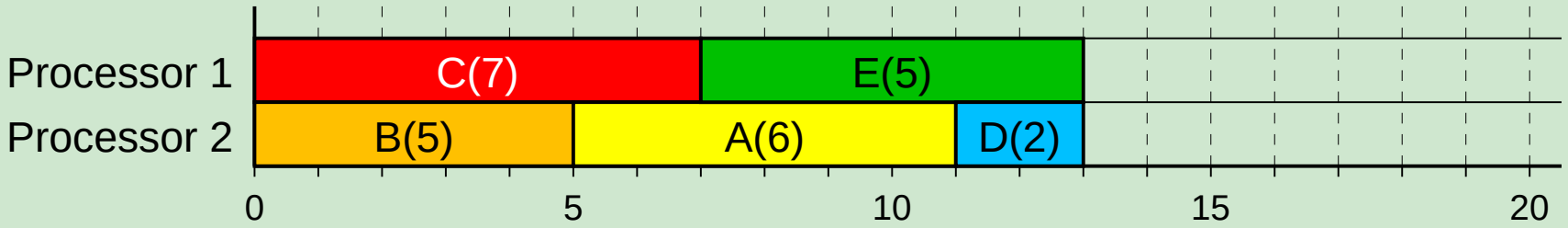

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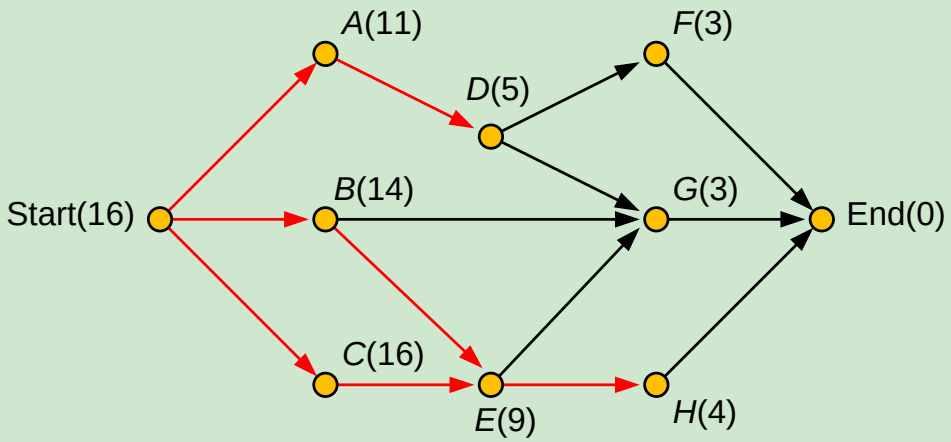

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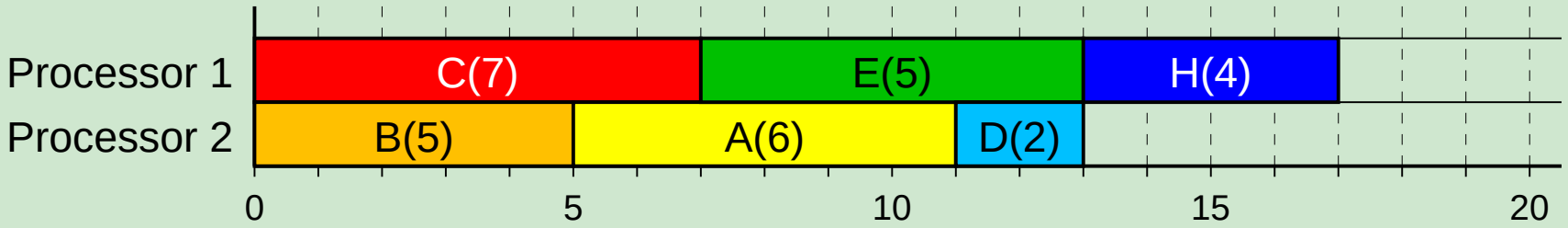

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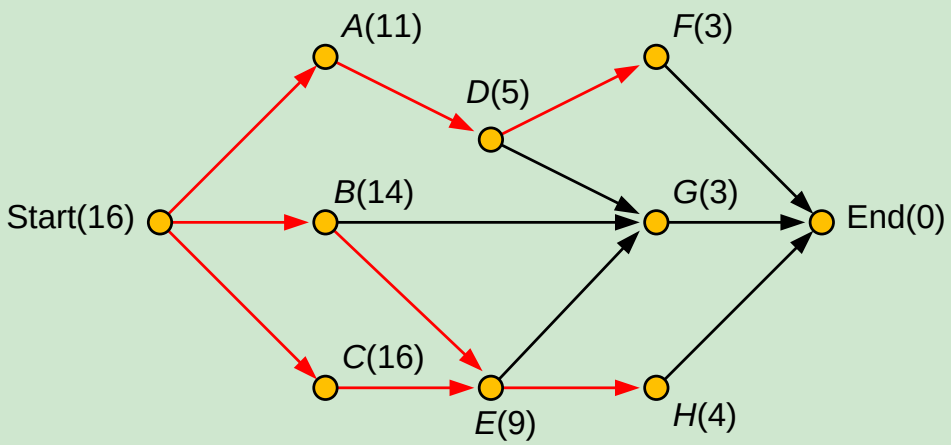

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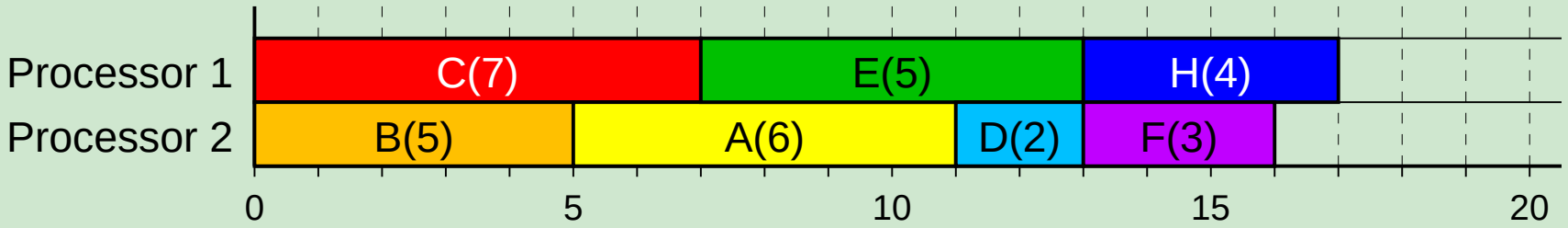

Example

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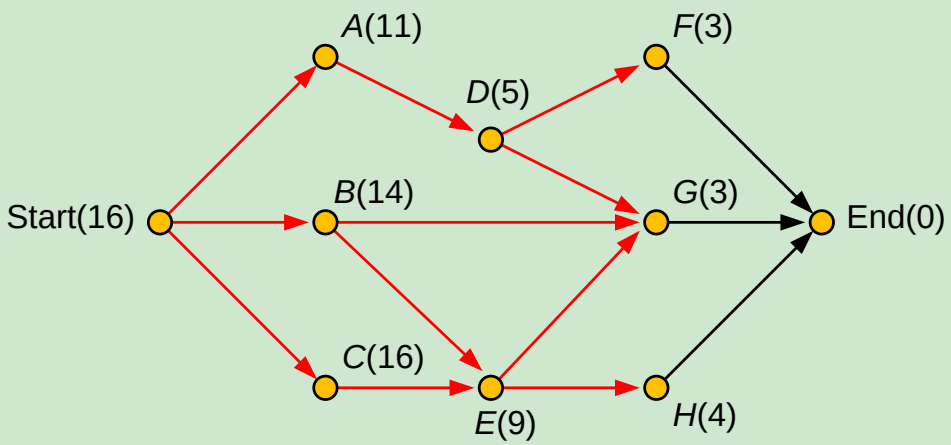

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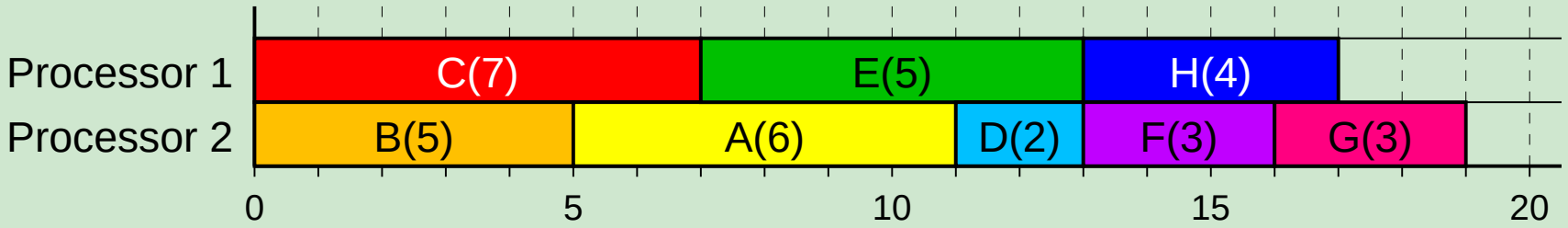

Example

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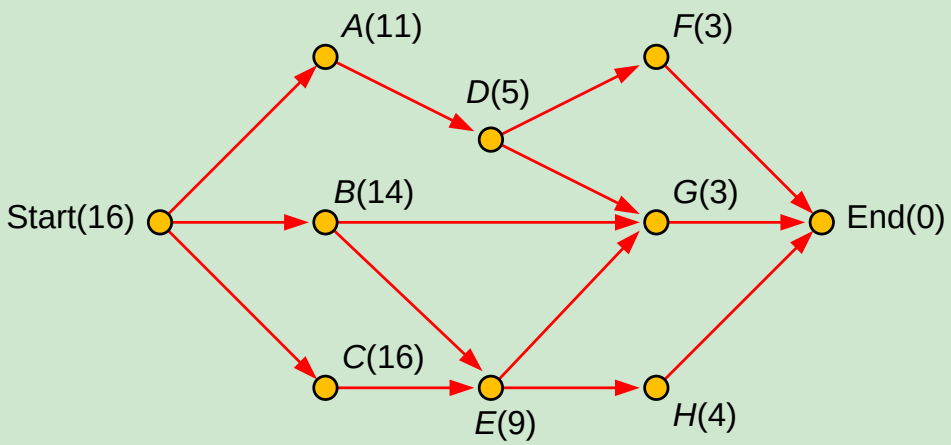

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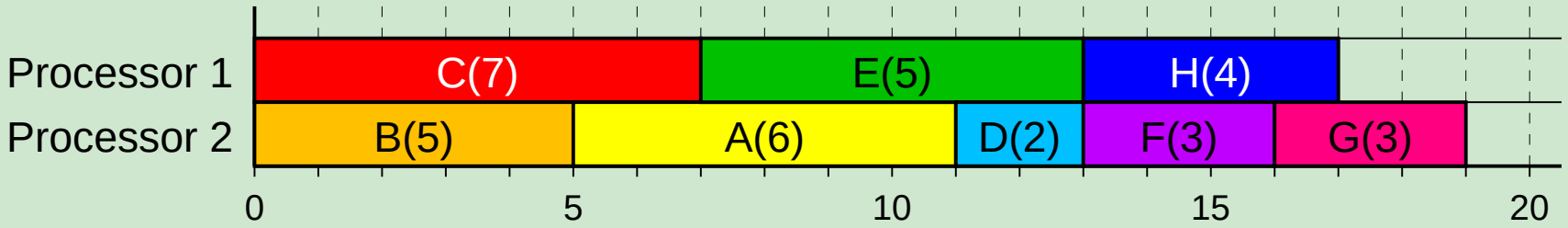

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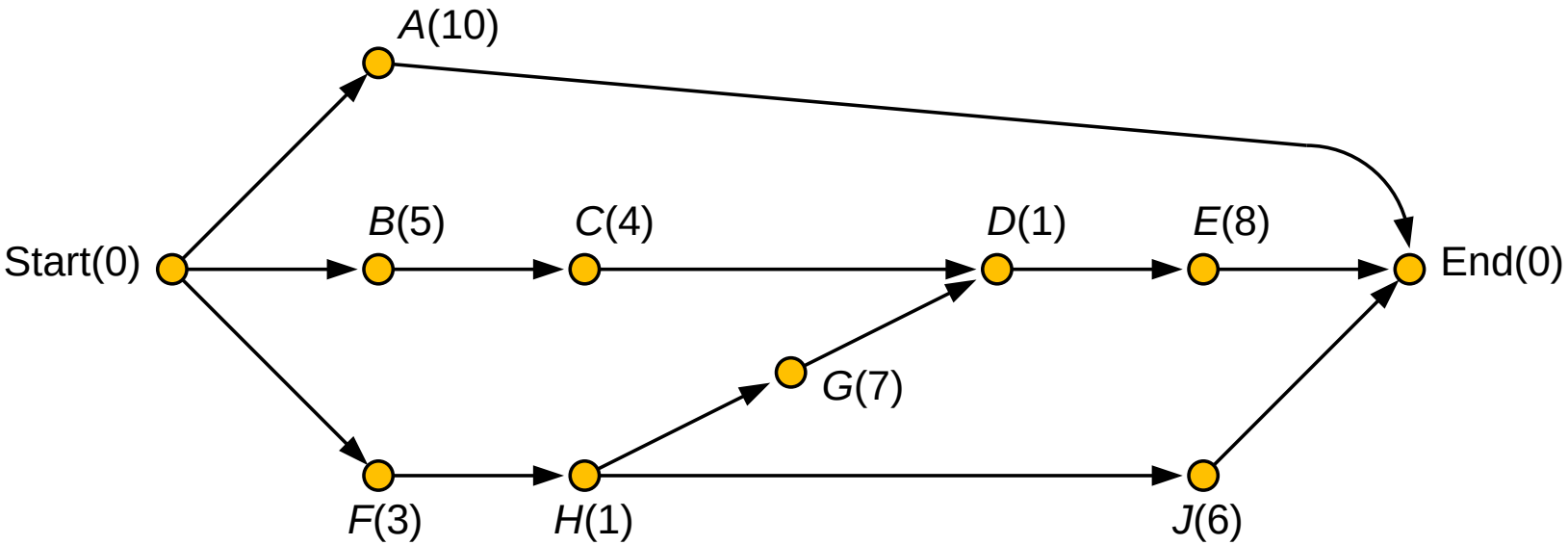
Example (Exercise 55)

Task	Precedencent Tasks	Processing Time
A		10
B		5
C	B	4
D	C, G	1
E	D	8
F		3
G	H	7
H	F	1
J	H	6

Example (Exercise 55)



Schedule the tasks using 2 processors.

Example (Exercise 55)

Task	Precedencent Tasks	Processing Time	Critical Time
A		10	
B		5	
C	B	4	
D	C, G	1	
E	D	8	
F		3	
G	H	7	
H	F	1	
J	H	6	

Example (Exercise 55)

Task	Precedencent Tasks	Processing Time	Critical Time
A		10	10
B		5	
C	B	4	
D	C, G	1	
E	D	8	
F		3	
G	H	7	
H	F	1	
J	H	6	

Example (Exercise 55)

Task	Precedencent Tasks	Processing Time	Critical Time
A		10	10
B		5	
C	B	4	
D	C, G	1	
E	D	8	8
F		3	
G	H	7	
H	F	1	
J	H	6	

Example (Exercise 55)

Task	Precedencent Tasks	Processing Time	Critical Time
A		10	10
B		5	
C	B	4	
D	C, G	1	
E	D	8	8
F		3	
G	H	7	
H	F	1	
J	H	6	6

Example (Exercise 55)

Task	Precedencent Tasks	Processing Time	Critical Time
A		10	10
B		5	
C	B	4	
D	C, G	1	9
E	D	8	8
F		3	
G	H	7	
H	F	1	
J	H	6	6

Example (Exercise 55)

Task	Precedencent Tasks	Processing Time	Critical Time
A		10	10
B		5	
C	B	4	13
D	C, G	1	9
E	D	8	8
F		3	
G	H	7	
H	F	1	
J	H	6	6

Example (Exercise 55)

Task	Precedencent Tasks	Processing Time	Critical Time
A		10	10
B		5	
C	B	4	13
D	C, G	1	9
E	D	8	8
F		3	
G	H	7	16
H	F	1	
J	H	6	6

Example (Exercise 55)

Task	Precedencent Tasks	Processing Time	Critical Time
A		10	10
B		5	
C	B	4	13
D	C, G	1	9
E	D	8	8
F		3	
G	H	7	16
H	F	1	17
J	H	6	6

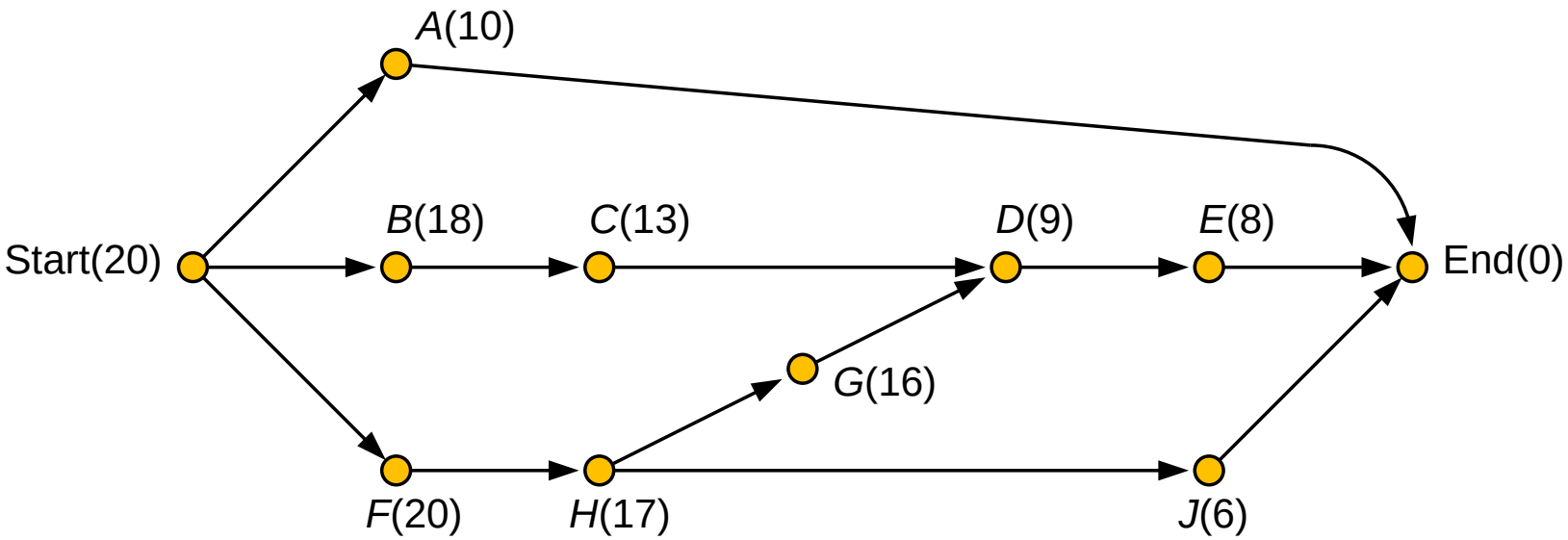
Example (Exercise 55)

Task	Precedencent Tasks	Processing Time	Critical Time
A		10	10
B		5	18
C	B	4	13
D	C, G	1	9
E	D	8	8
F		3	
G	H	7	16
H	F	1	17
J	H	6	6

Example (Exercise 55)

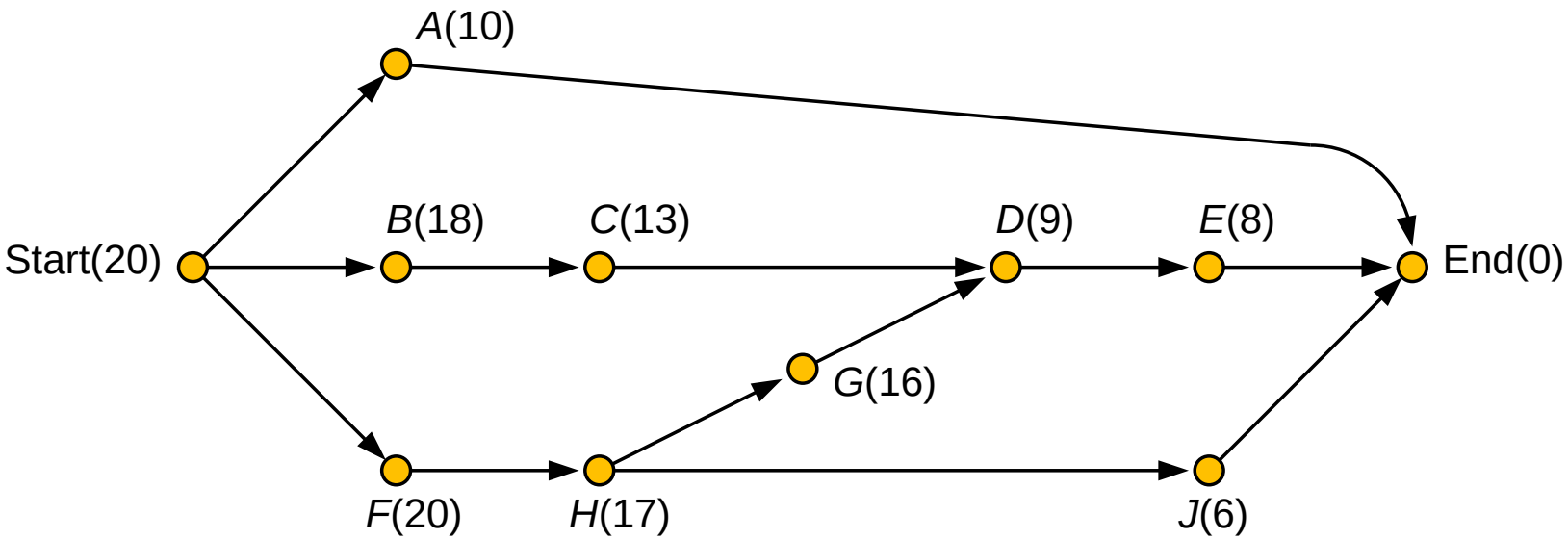
Task	Precedencent Tasks	Processing Time	Critical Time
A		10	10
B		5	18
C	B	4	13
D	C, G	1	9
E	D	8	8
F		3	20
G	H	7	16
H	F	1	17
J	H	6	6

Example (Exercise 55)



Schedule the tasks using 2 processors.

Example (Exercise 55)



Schedule the tasks using 3 processors.

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- Chapter 8: Exercises 49, 52, 53, 56.