The Five-Number Summary
Lecture 16
Sections 5.3.1 - 5.3.3

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Outline

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Exercise 5.3, p. 311.

A professor teaches two statistics classes. The morning class has 25 students, and their average on the first test was 82. The evening class has 15 students, and their average on the same test was 74. What is the average on this test if the professor combines the scores for both classes?

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$$
\begin{aligned}
\text{avg.} &= \frac{25(82) + 15(74)}{25 + 15} \\
&= \frac{2050 + 1110}{40} \\
&= \frac{3160}{40} \\
&= 79.
\end{aligned}
$$

Outline

Percentiles and Quartiles

Definition (p^{th} Percentile)

The pth percentile of a set of numbers is a number that divides the lower p% of the numbers from the rest.

Definition (1st Quartile)

The 1st quartile, denoted Q_1, of a set of numbers is the 25th percentile.

Definition (3rd Quartile)

The 3rd quartile, denoted Q_3, of a set of numbers is the 75th percentile.

- To find the quartiles, first find the position of the median.
- Then the 1st quartile is the median of all the numbers that are below that position.
- The 3rd quartile is the median of all the numbers that are above that position.

Example

Example (Quartiles)

- Find the median and quartiles of the following sample.

5, 8, 10, 15, 17, 19, 20, 24, 25, 30, 32

Example

Example (Quartiles)

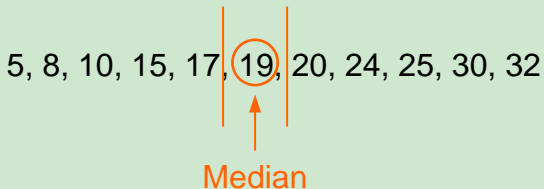- Find the median and quartiles of the following sample.

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Median

Example

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$$Q_1 \qquad \text{Median} \qquad Q_3$$

Outline

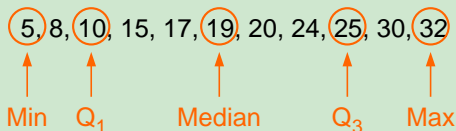## Definition (Five-Number Summary)

The five-number summary of a set of numbers consists of the five quantities

- Minimum
- 1st quartile
- Median
- 3rd quartile
- Maximum

- These five numbers divide the set of numbers into four groups of equal size, each containing one-fourth of the set.

Example

Example (Five-Number Summary)

- The five-number summary of the previous sample is
  - Min$= 5$.
  - $Q_1 = 10$.
  - Med$= 19$.
  - $Q_3 = 25$.
  - Max$= 32$.

5, 8, 10, 15, 17, 19, 20, 24, 25, 30, 32

Min Q_1 Median Q_3 Max

Practice

- Find the five-number summary of the Plant B data from Exercise 4.36, page 262.

| 27 | 29 | 31 | 32 | 32 | 33 | 33 | 34 | 37 | 44 |
| 46 | 51 | 56 | 56 | 57 | 58 | 58 | 59 | 62 | 63 |

Outline

TI-83 Five-Number Summary

TI-83 Five-Number Summary

- Follow the same procedure that was used to find the mean.
- When the list of statistics appears, scroll down to the ones labeled

$$\mathtt{minX, Q1, Med, Q3, maxX.}$$

- They are the five-number summary.
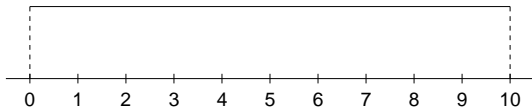
TI-83 Five-Number Summary

TI-83 Five-Number Summary

- Use the TI-83 to find the five-number summary of the rainfall data

5.94	1.11	9.52	0.08	6.14	8.68
2.93	2.03	3.60	14.71	4.01	0.85
6.89	11.07	4.42	3.41	2.85	2.56
1.92	5.15	1.58	4.44	0.77	4.76
1.15	3.02	1.73	2.60	2.56	10.01

Five-Number Summaries and Distributions

- If the distribution were uniform from 0 to 10, what would be the five-number summary?

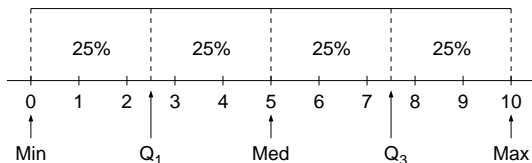- If the distribution were uniform from 0 to 10, what would be the five-number summary?

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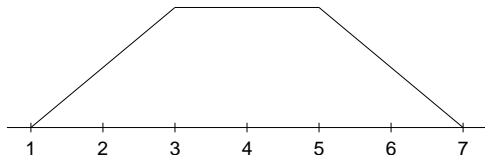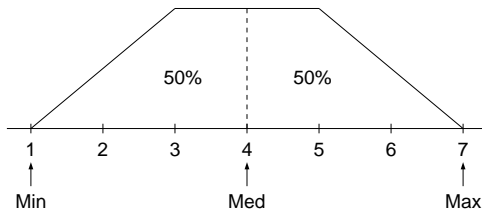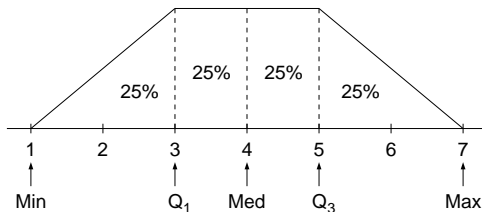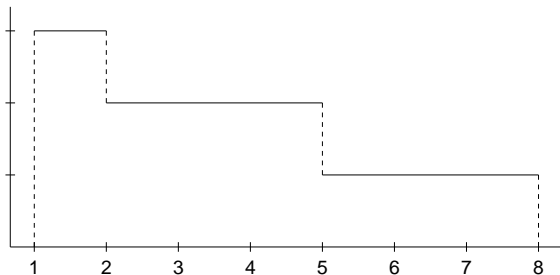
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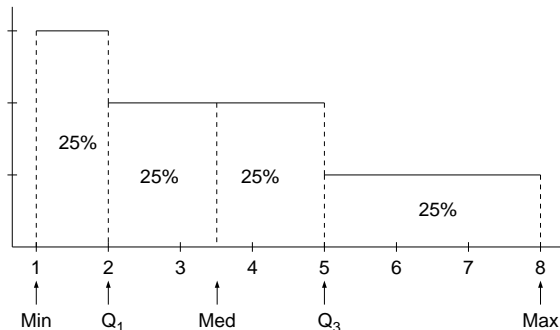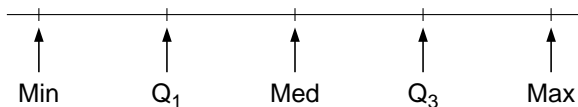
Five-Number Summaries and Distributions

- Where would the median and quartiles be in this non-symmetric non-uniform distribution?

Five-Number Summaries and Distributions

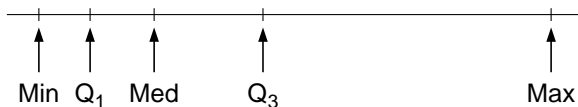- Where would the median and quartiles be in this non-symmetric non-uniform distribution?
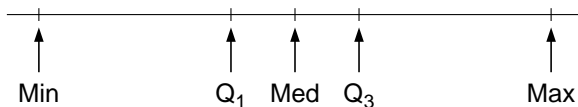
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Five-Number Summaries and Distributions

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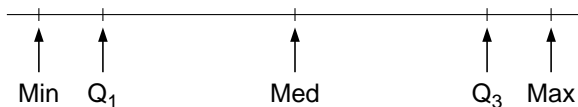Five-Number Summaries and Distributions

- Describe the distribution.

Outline

The Interquartile Range

Definition (Interquartile Range)

The interquartile range, denoted IQR, is the difference between Q_3 and Q_1.

- The IQR is a commonly used measure of spread, or variability.
- Like the median, it is not affected by extreme outliers.

The IQR

Example (IQR)

- The IQR of

$$5, 8, 10, 15, 17, 19, 20, 24, 25, 30, 32$$

  is

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 25 - 10 \\ &= 15 \end{aligned}$$

The IQR

Practice

- Find the IQR of the Plant B data

27	29	31	32	32	33	33	34	37	44
46	51	56	56	57	58	58	59	62	63

and use it to describe the sample.

- Use the stem-and-leaf display to find a five-number summary of the Plant B data.

Stem	Leaf
2	7 9
3	1 2 2 3 3 4 7
4	4 6
5	1 6 6 7 8 8 9
6	2 3

- Note: 1|2 means 12.

Salaries of School Board Chairmen

Practice

- Find the five-number summary of the following salaries of school board chairmen.

County/City	Salary	County/City	Salary
Henrico	20,000	Caroline	5,000
Chesterfield	18,711	Louisa	4,921
Richmond	11,000	Powhatan	4,800
Hanover	11,000	Hopewell	4,500
Petersburg	8,500	Charles City	4,500
Sussex	7,000	Prince George	3,750
New Kent	6,500	Cumberland	3,600
Goochland	5,500	King & Queen	3,000
Dinwiddie	5,120	King William	2,400
Colonial Hgts	5,100		

Outline

Excel's Definition of Percentile

Definition (Excel's p^{th} percentile)

Excel's p^{th} percentile of a set of numbers is the number whose rank (position) is given by

$$r = 1 + \left(\frac{p}{100}\right)(n-1).$$

If r is not a whole number, then interpolate between values.

- Microsoft's Excel uses a definition of the pth percentile that is based on the *gaps* between the numbers rather than on the numbers themselves.

Outline

Assignment

Homework

- Read Section 5.3.1 - 5.3.2, pages 312 - 315.
- Work Example 5.4, page 314, as an exercise.