Residual Analysis and Outliers
Sections 5.5, 5.6
Lecture 15

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Outline

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Introduction

- How do we know that a linear regression model is the best choice?

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- How many would you like?

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- How do we know that a linear regression model is the best choice?
- What other types of regression are there?
- There are many other types.
- How many would you like?
- The linear model is by far the simplest, but it is not the only choice.

TI-83 - Nonlinear Regression

TI-83 Nonlinear Regression

- The TI-83 will do a variety of *nonlinear* regressions.
- Press `STAT > CALC`.
- The list includes
  - `LinReg` - Linear regression:

$$\hat{y} = a + bx.$$

  - `QuadReg` - Quadratic regression:

$$\hat{y} = ax^2 + bx + c.$$

  - `CubicReg` - Cubic regression:

$$\hat{y} = ax^3 + bx^2 + cx + d.$$

TI-83 Nonlinear Regression

- And...
  - `QuartReg` - Quartic regression:

$$\hat{y} = ax^4 + bx^3 + cx^2 + dx + e.$$

  - `LnReg` - Logarithmic regression:

$$\hat{y} = a + b \ln x.$$

  - `ExpReg` - Exponential regression:

$$\hat{y} = ab^x.$$

TI-83 Nonlinear Regression

- And...
  - `PwrReg` - Power regression:

$$\hat{y} = ax^b.$$

  - `Logistic` - Logistic regression:

$$\hat{y} = \frac{c}{1 + ae^{-bx}}.$$

  - `SinReg` - Sinusoidal regression:

$$\hat{y} = a\sin(bx + c) + d.$$

Outline

The Appropriateness of the Linear Model

- We can learn a bit about the nature of the model by examining the residuals.
- This is called residual analysis.
- First, we need to find the residuals

$$\text{residual} = y - \hat{y}_i.$$

- Then we draw a scatterplot of x versus the residuals and see whether there is a pattern.

The Appropriateness of the Linear Model

- To do this on the TI-83, first find the predicted values $\hat{y}$ and store them in $\texttt{L}_3$:

$$\texttt{Y}_1(\texttt{L}_1) \longrightarrow \texttt{L}_3$$

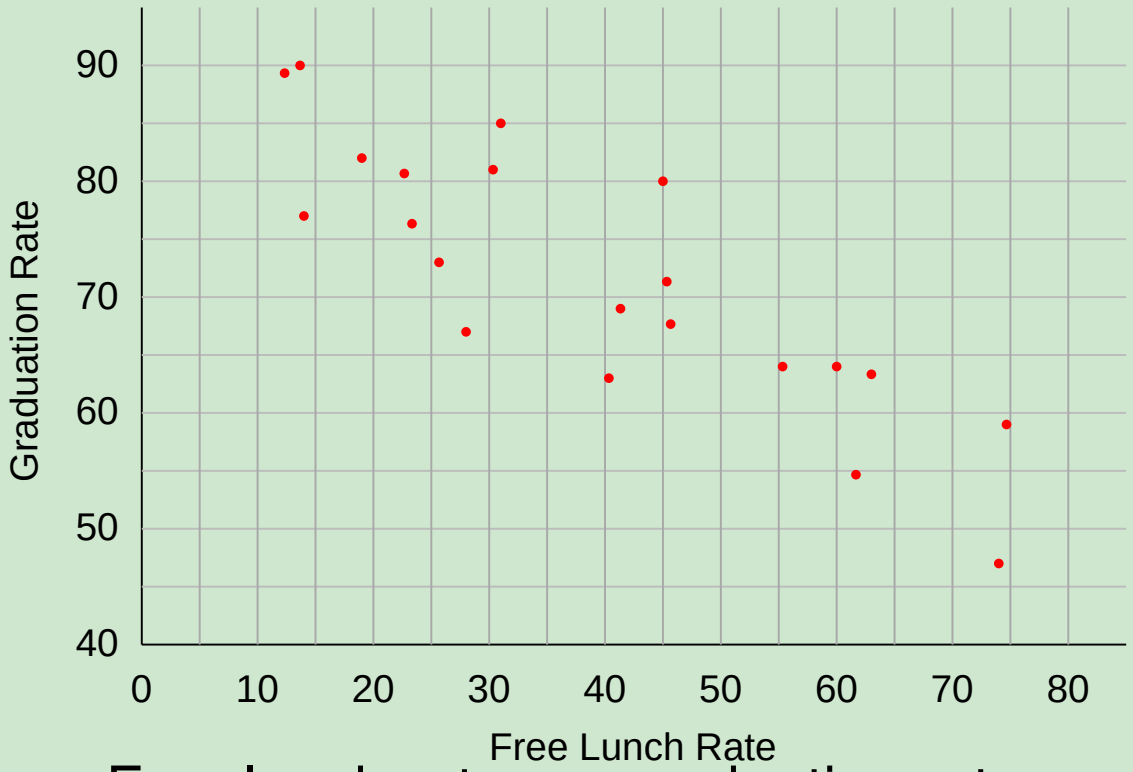
- Then find the residuals and store them in $\texttt{L}_4$:

$$\texttt{L}_2 - \texttt{L}_3 \longrightarrow \texttt{L}_4$$

- Then draw a scatterplot of $\texttt{L}_1$ (x) versus $\texttt{L}_4$ (residuals).

The Residual Plot

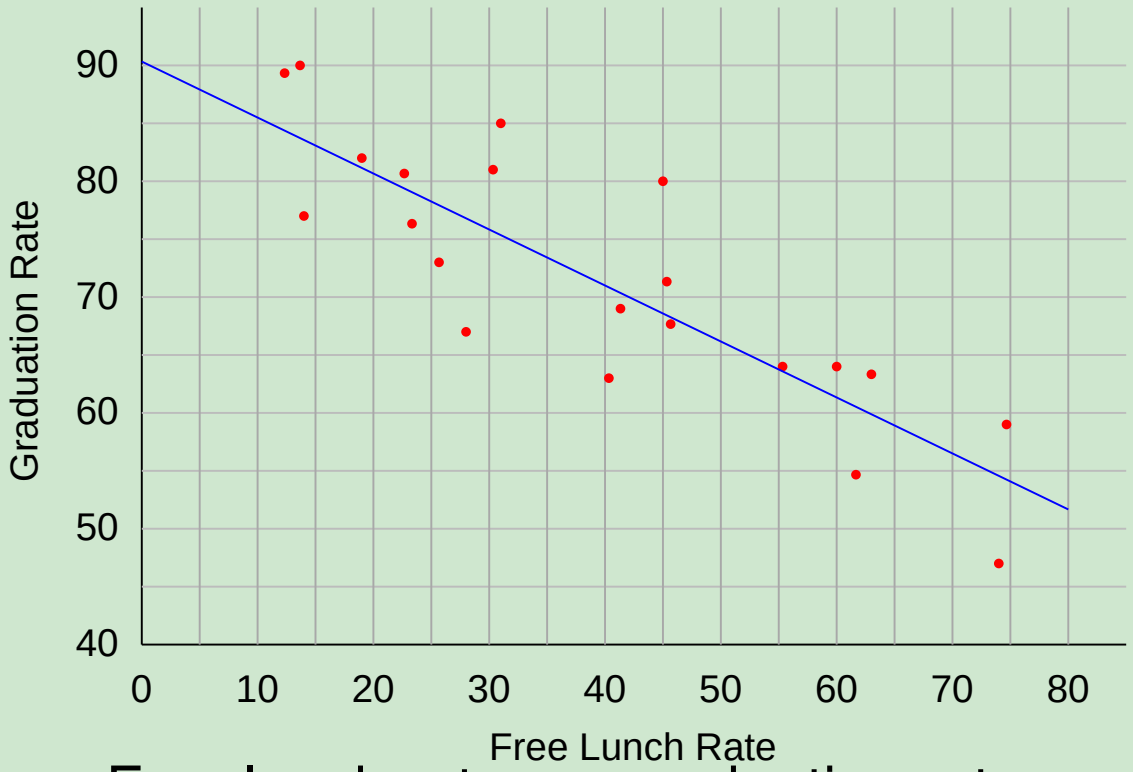
Example (Residual Plots)



Free lunch rate vs. graduation rate

The Residual Plot

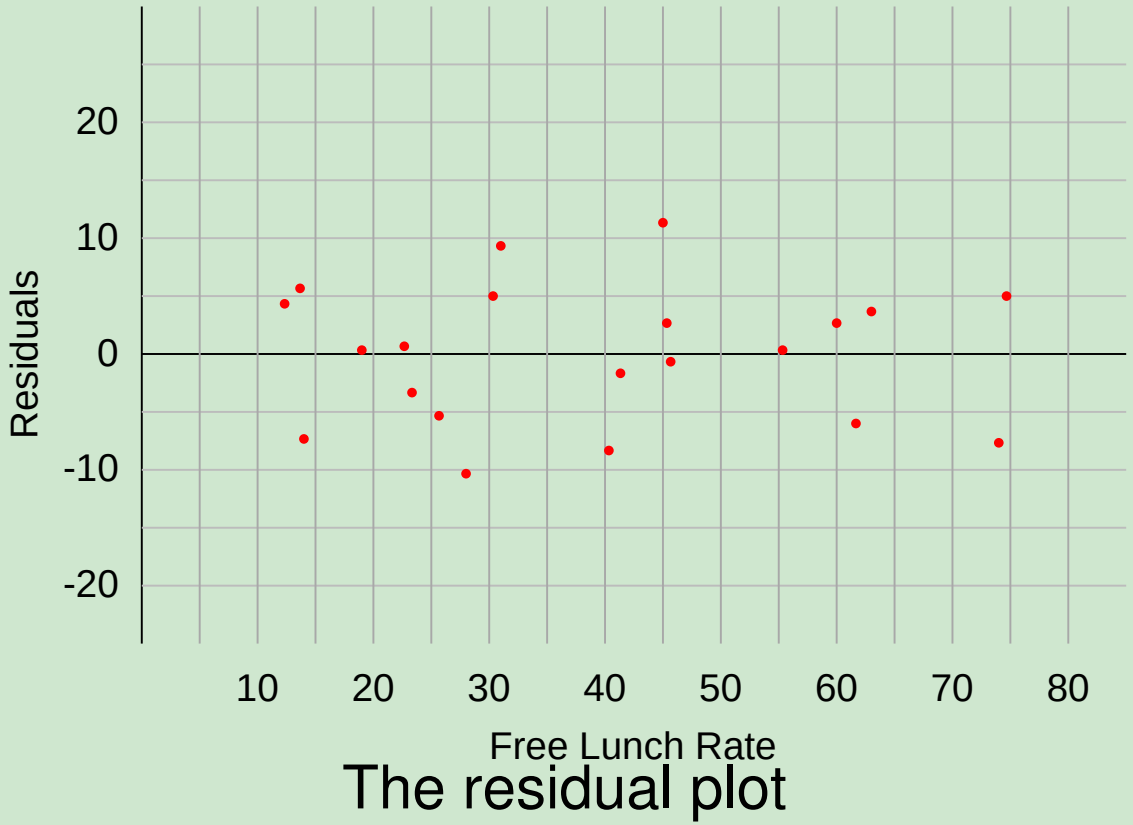
Example (Residual Plots)



Free lunch rate vs. graduation rate

The Residual Plot

Example (Residual Plots)



The residual plot

The Appropriateness of the Linear Model

- If the residual plot shows no clear pattern, but just a big blob of points, then the linear model is appropriate.
- On the other hand, if the residual plot shows a distinct curvature, or any other distinct pattern, then the linear model may not be appropriate.

Outline

A Nonlinear Model

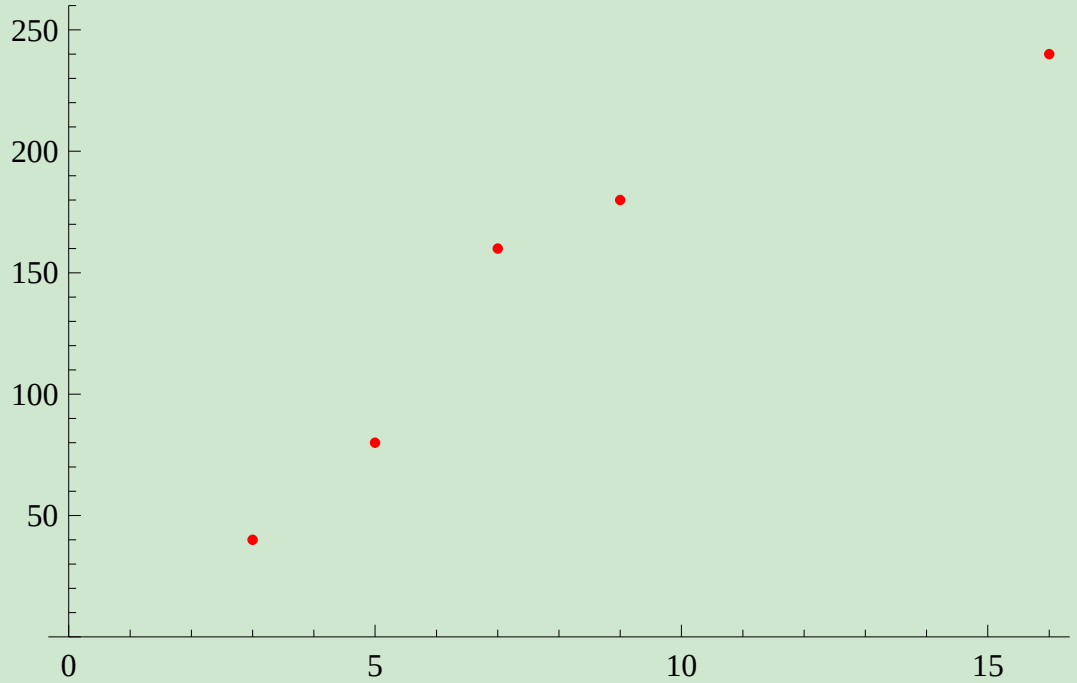
Example (A Nonlinear Model)

- Consider the following data.

x	y
3	40
5	80
7	160
9	180
16	240

A Nonlinear Model

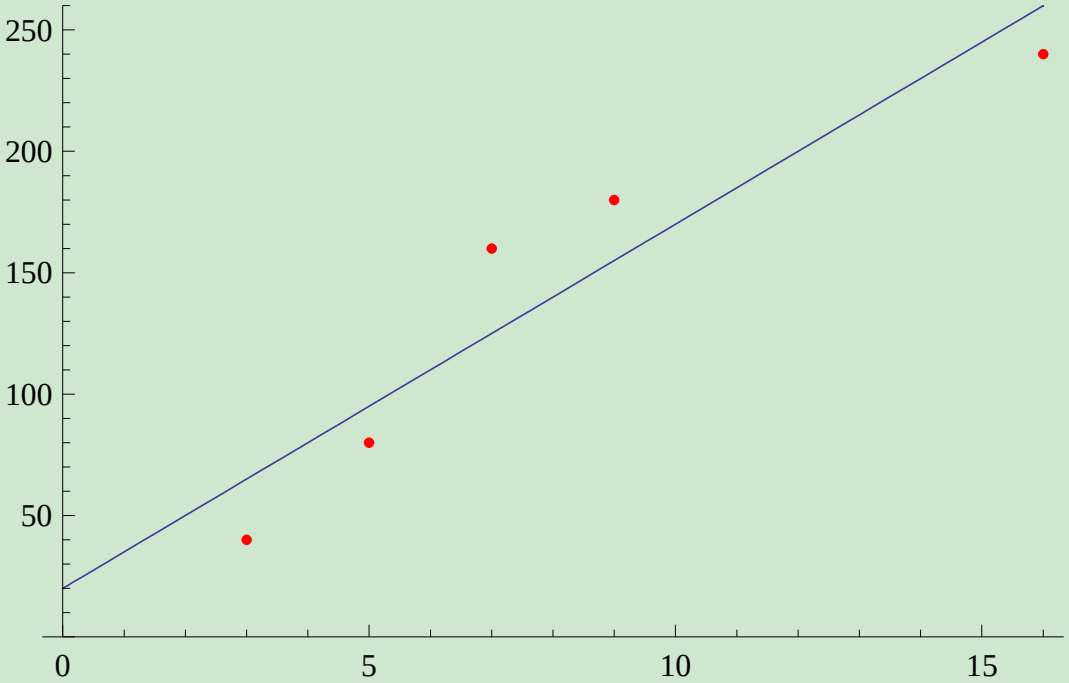
Example (A Nonlinear Model)



The scatterplot

A Nonlinear Model

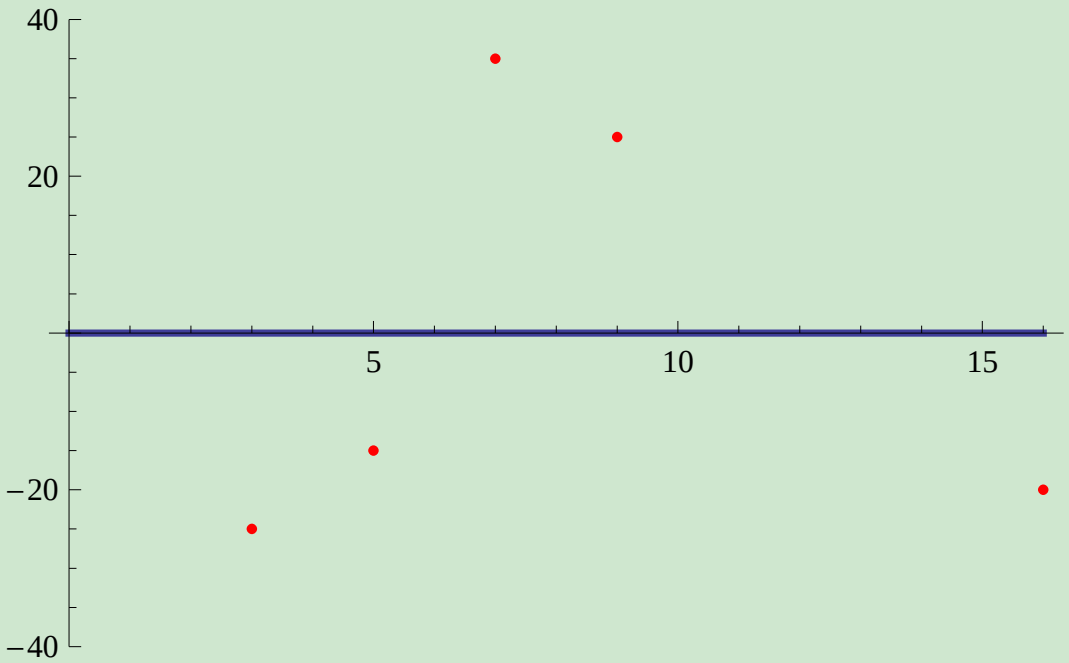
Example (A Nonlinear Model)



The regression line

A Nonlinear Model

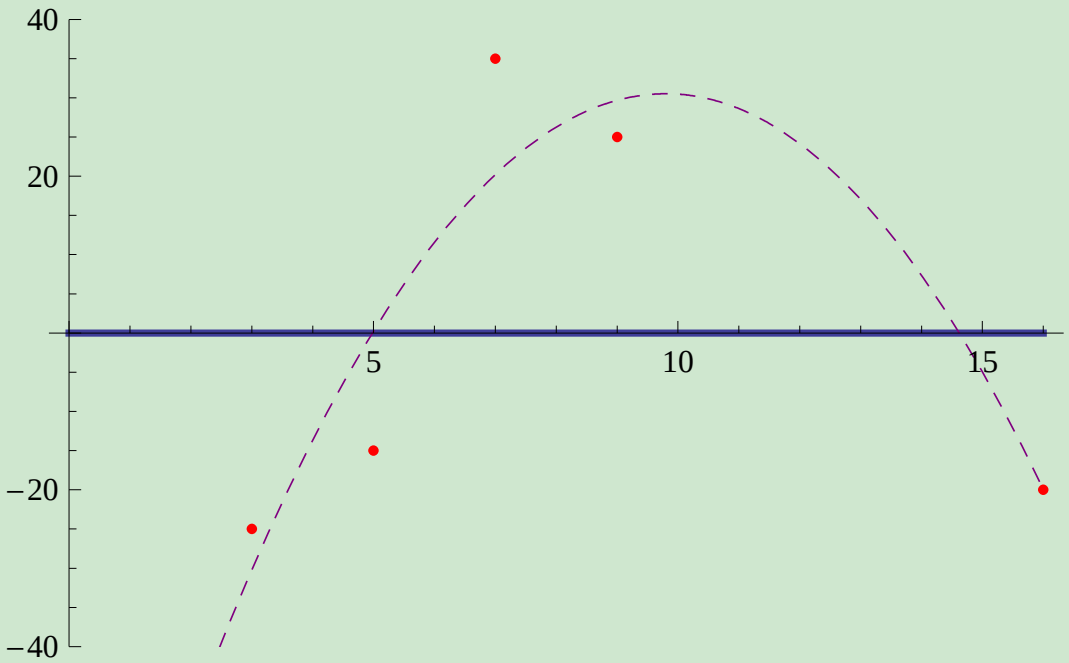
Example (A Nonlinear Model)



The residual plot

A Nonlinear Model

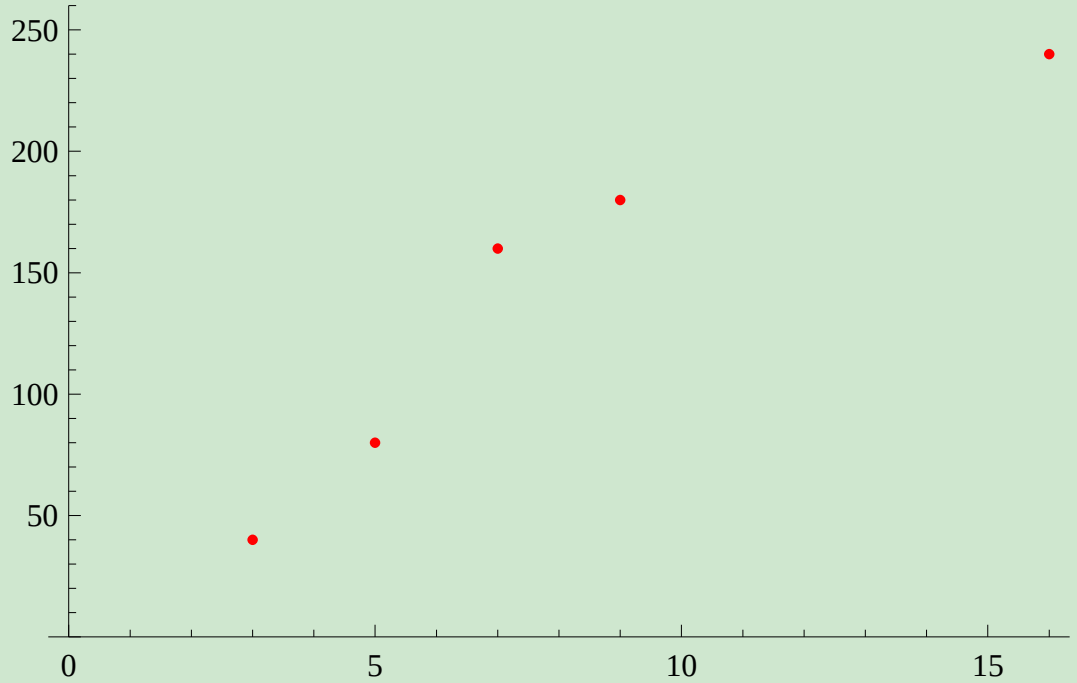
Example (A Nonlinear Model)



The residual plot

A Nonlinear Model

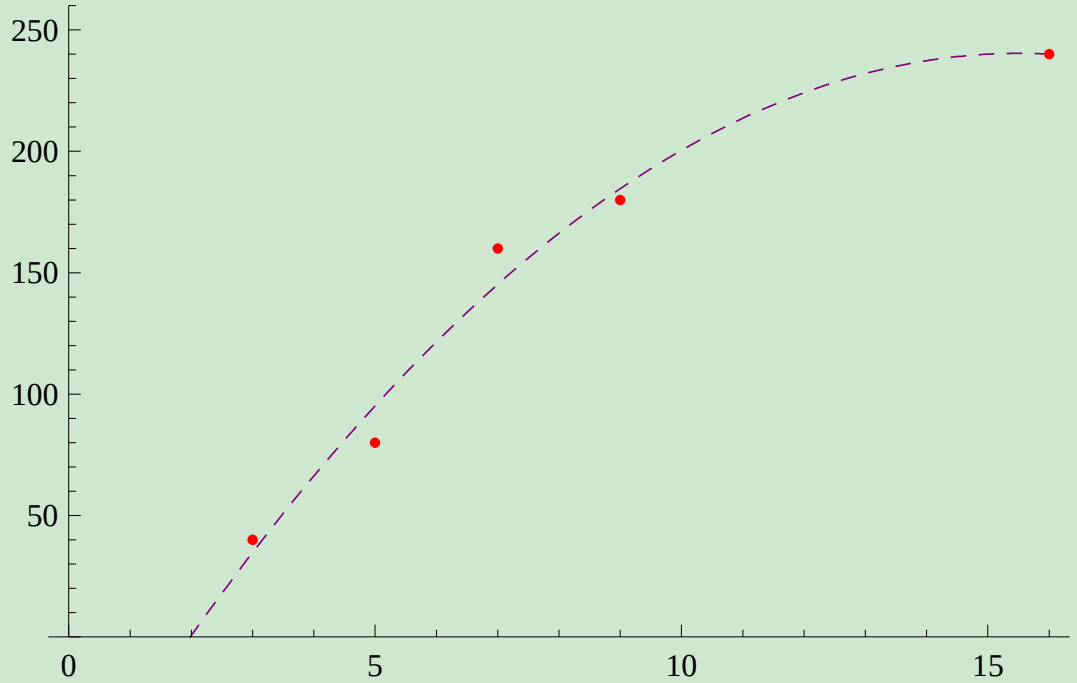
Example (A Nonlinear Model)



Quadratic regression

A Nonlinear Model

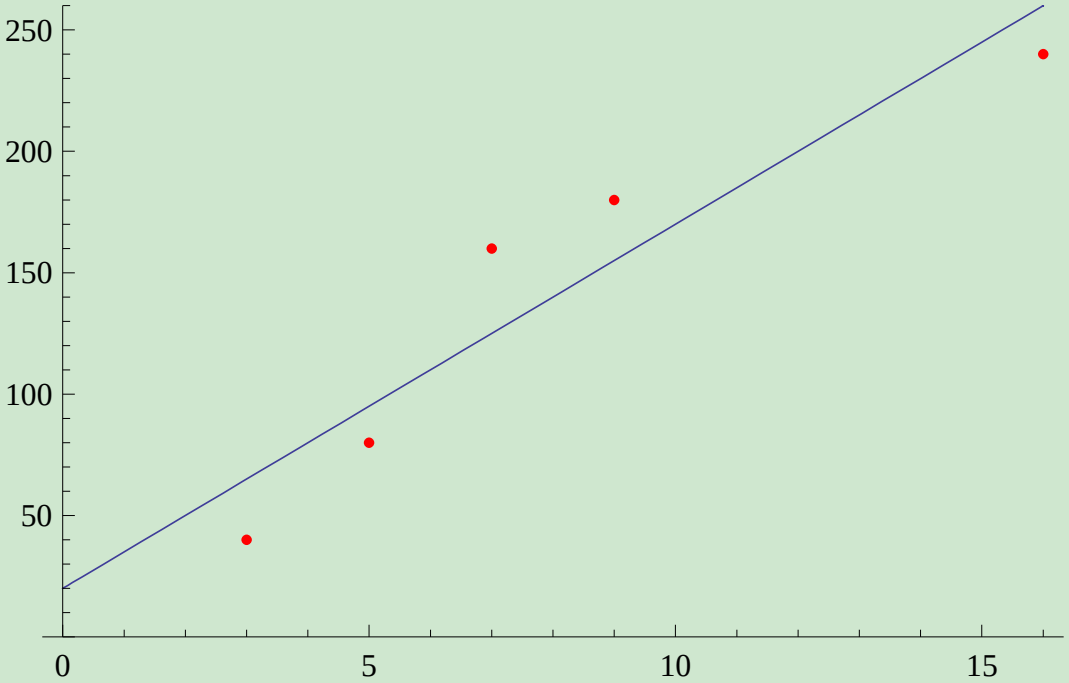
Example (A Nonlinear Model)



Quadratic regression

A Nonlinear Model

Example (A Nonlinear Model)



Linear regression

Outline

Outliers

Definition (Outlier)

An outlier is a point with an unusually large residual (e.g., at least 2.5 standard deviations from the mean).

Definition (Influential Point)

An influential point is a point that exerts a inordinate influence on the regression line.

Outliers

- An outlier may or may not be influential.
- An influential point may or may not be an outlier.

Outliers and Influential Points

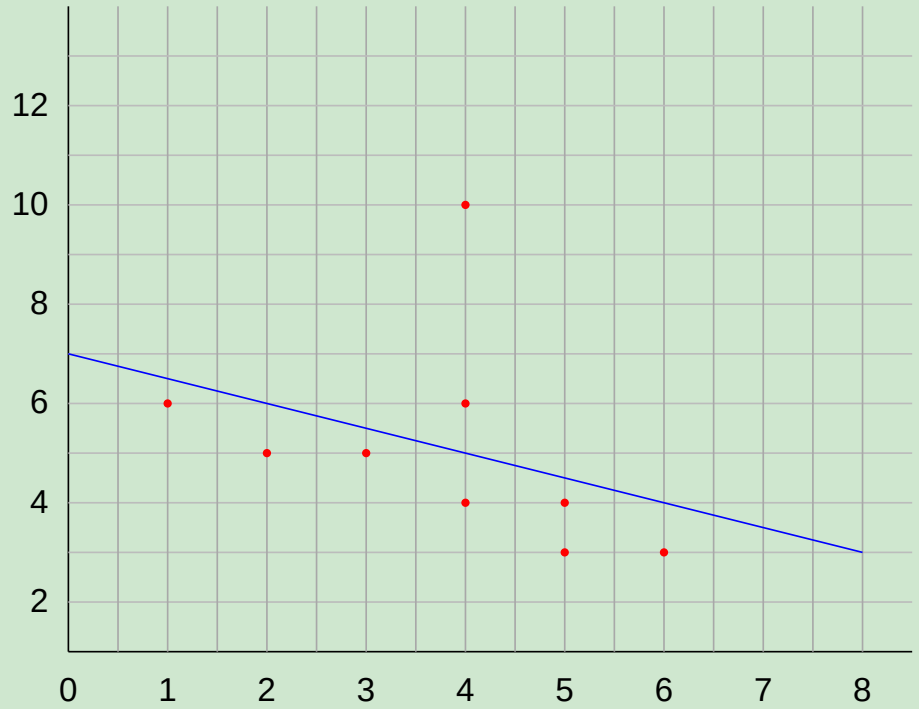
Example (Outliers and Influential Points)

x	y
1	6
2	5
3	5
4	6
4	4
4	10
5	3
5	4
6	3

- Consider the above data.

Outliers and Influential Points

Example (Outliers and Influential Points)



The scatterplot

Outliers and Influential Points

Example (Outliers and Influential Points)

x	y	$\hat{y}$	$y - \hat{y}$
1	6		
2	5		
3	5		
4	6		
4	4		
4	10		
5	3		
5	4		
6	3		

- The regression line is $\hat{y} = 7.0 - 0.5x$.

Outliers and Influential Points

Example (Outliers and Influential Points)

x	y	$\hat{y}$	$y - \hat{y}$
1	6	6.5	
2	5	6.0	
3	5	5.5	
4	6	5.0	
4	4	5.0	
4	10	5.0	
5	3	4.5	
5	4	4.5	
6	3	4.0	

- The regression line is $\hat{y} = 7.0 - 0.5x$.

Outliers and Influential Points

Example (Outliers and Influential Points)

x	y	$\hat{y}$	$y - \hat{y}$
1	6	6.5	-0.5
2	5	6.0	-1.0
3	5	5.5	-0.5
4	6	5.0	1.0
4	4	5.0	-1.0
4	10	5.0	5.0
5	3	4.5	-1.5
5	4	4.5	-0.5
6	3	4.0	-1.0

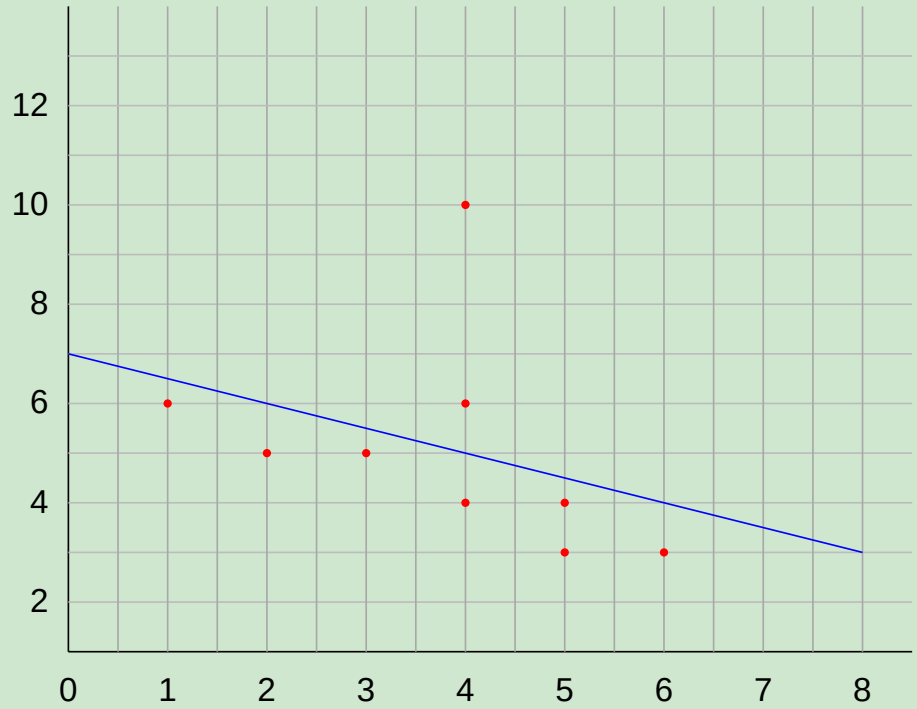
- The regression line is $\hat{y} = 7.0 - 0.5x$.

Outliers and Influential Points

- The mean residual is 0.0 (always) and the standard deviation of these residuals is 2.0.
- Thus, the residual 5.0 is 2.5 standard deviations above the mean, an outlier.
- But, is the point $(4, 10)$ influential?
- Remove it and see what the effect is.

Outliers and Influential Points

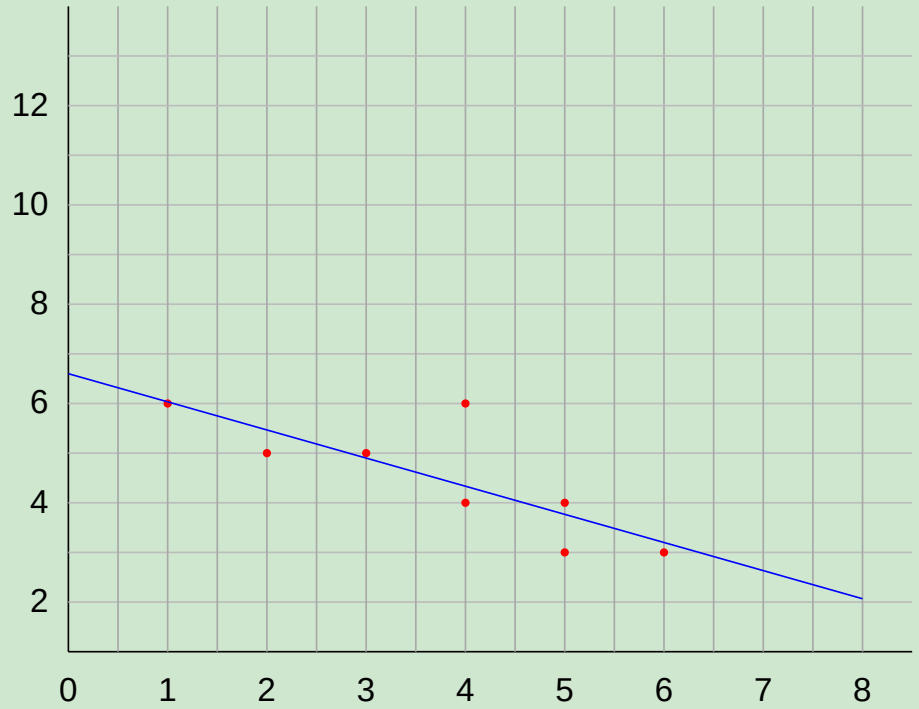
Example (Outliers and Influential Points)



Including the point $(4, 10)$

Outliers and Influential Points

Example (Outliers and Influential Points)



Excluding the point $(4, 10)$

Outliers and Influential Points

- The regression line of the remaining points is

$$\hat{y} = 6.615 - 0.564x.$$

- This is nearly the same as

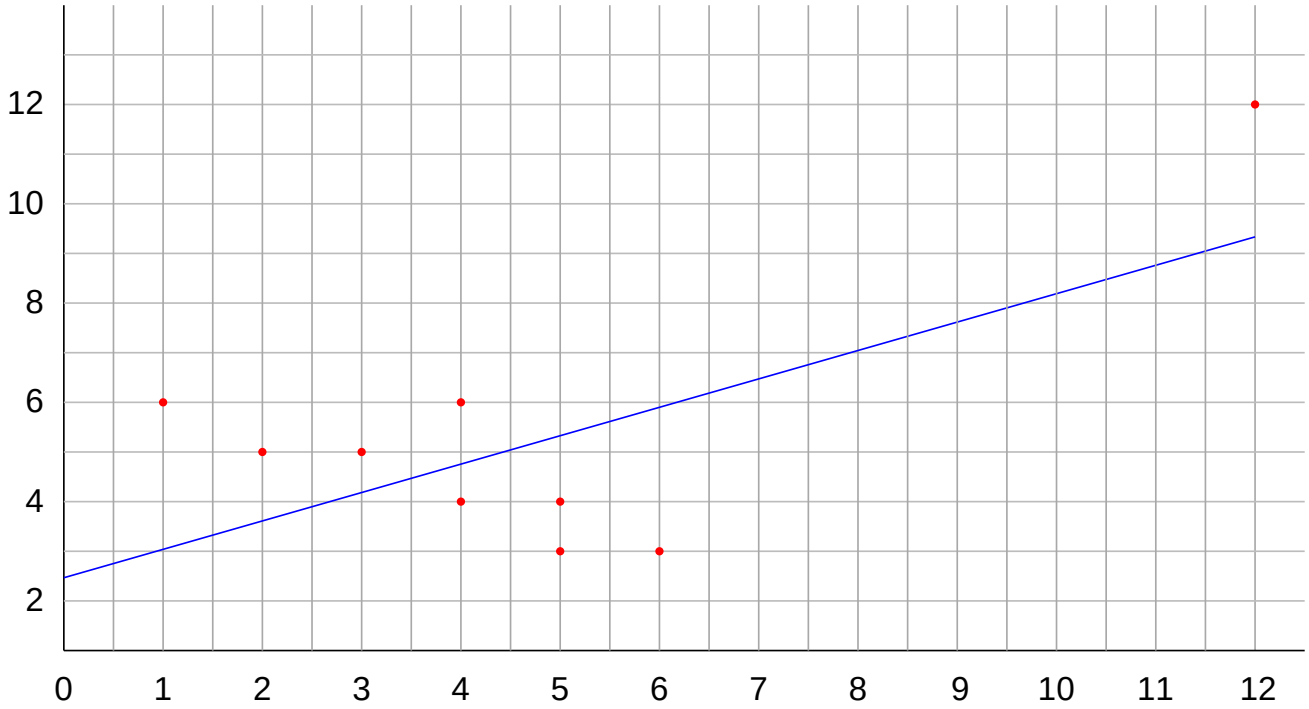
$$\hat{y} = 7.0 - 0.5x.$$

Outliers and Influential Points

- Now change the point $(4, 10)$ to the point $(12, 12)$.

x	y
1	6
2	5
3	5
4	6
4	4
5	3
5	4
6	3
12	12

Outliers and Influential Points



Is $(12, 12)$ an outlier?

Outliers and Influential Points

- The regression line including $(12, 12)$ is

$$\hat{y} = 2.767 + 0.55x.$$

- Removing $(12, 12)$ changes it to

$$\hat{y} = 6.615 - 0.564x$$

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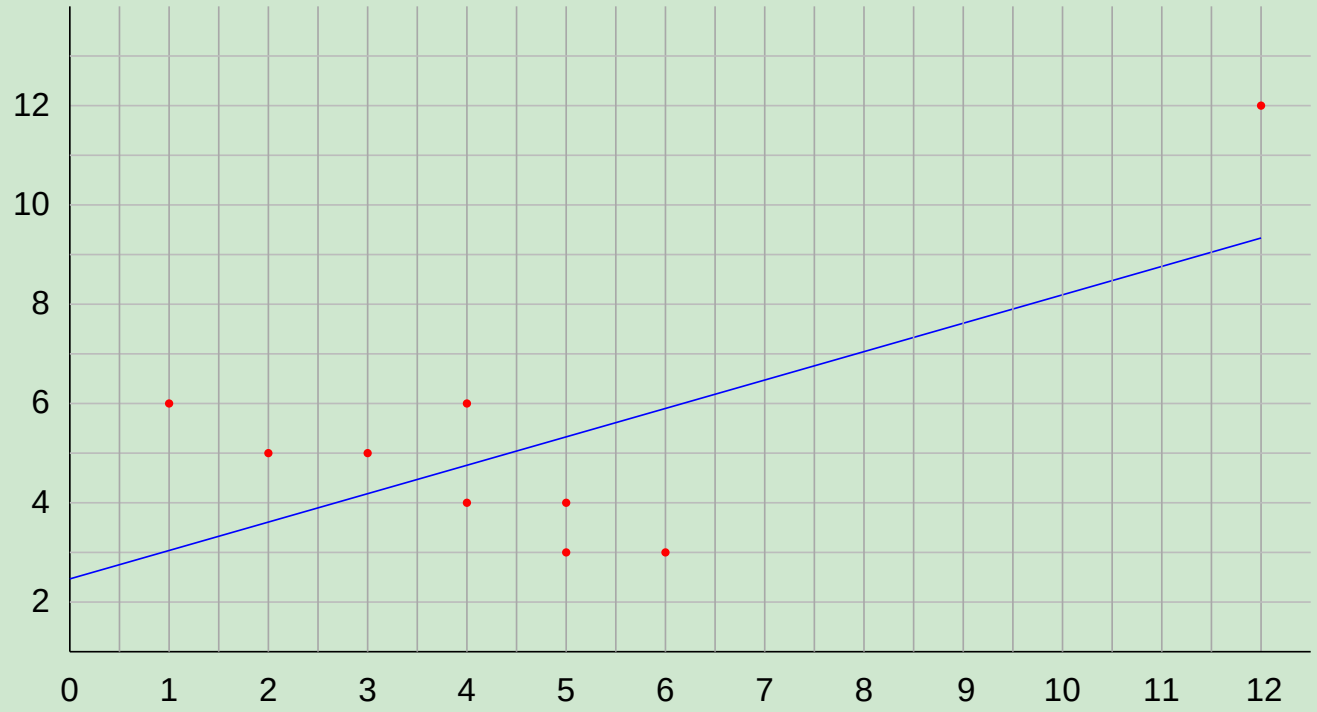
Outliers and Influential Points

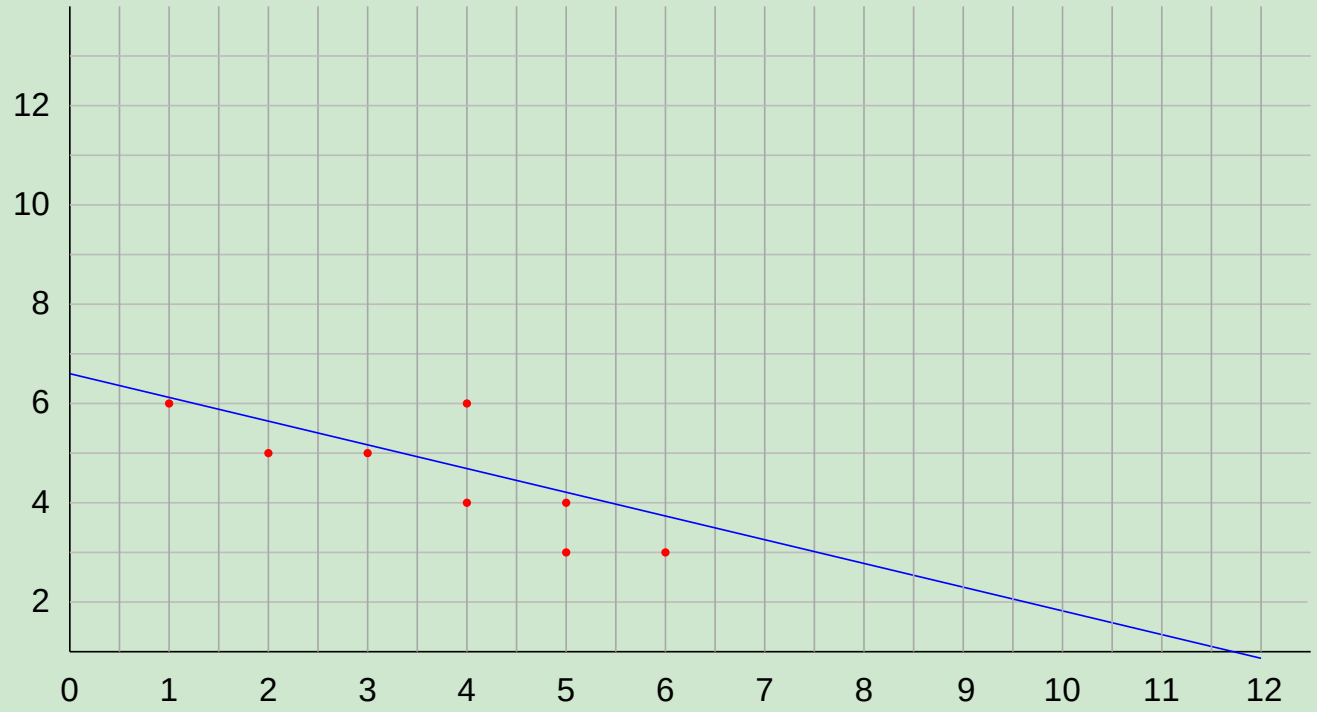
Example (Outliers and Influential Points)



Including the point $(12, 12)$

Outliers and Influential Points

Example (Outliers and Influential Points)



Excluding the point $(12, 12)$

Outliers and Influential Points

- Yet the residual of $(12, 12)$ is only 2.63.
- The standard deviation of the set of residuals is 2.12.
- $(12, 12)$ is only 1.24 standard deviations above the mean.
- Therefore, $(12, 12)$ is not an outlier, but it is influential.

Outline

Assignment

Assignment

- Read Sections 5.5, 5.6.
- Apply Your Knowledge: 8.
- Exercises 35(c), 42, 55, 61.