

Random Variables

Sections 12.7

Lecture 25

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Outline

- 1 Random Variables
 - The Uniform Distribution
 - A Non-uniform Distribution
 - Normal Distributions

- 2 Assignment

1 Random Variables

- The Uniform Distribution
- A Non-uniform Distribution
- Normal Distributions

2 Assignment

Random Variables

Definition (Random variable)

A **random variable** is a variable whose value is determined by the outcome of a random process.

Definition (Discrete random variable)

A **discrete random variable** is a random variable whose set of possible values is a discrete set.

Definition (Continuous random variable)

A **continuous random variable** is a random variable whose set of possible values is a continuous set.

Discrete Probability Density Functions

Definition (Discrete Probability Density Function)

A **discrete probability distribution function**, or **pdf**, for a discrete random variable X is function that is represented by a table of values of X paired with their probabilities.

Example

Example (Discrete Probability Density Function)

- Roll a pair of dice and let X be the sum of the two numbers.
- Describe the pdf of X .
- Draw the graph of the pdf.

Example

Example (Discrete Probability Density Function)

- One of four keys will unlock a door, but we do not know which key it is.
- So we try one key after another, randomly, but “without replacement.”
- Let X be the number of keys we try until we unlock the door.
- Describe the pdf of X . (Draw a tree diagram of the possibilities.)
- Draw the graph of the pdf.

Continuous Probability Density Functions

Definition (Continuous Probability Density Function)

A **continuous probability distribution function**, or **pdf**, for a continuous random variable X is a continuous function with the property that the area below the graph of the function between any two points a and b equals the probability that $a \leq X \leq b$.

- Remember,

AREA = PROPORTION = PROBABILITY

Outline

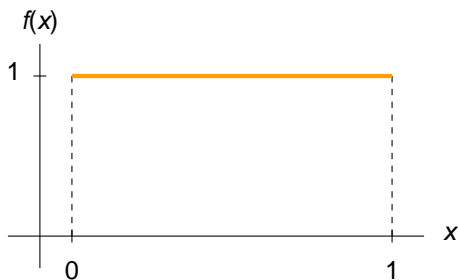
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Example

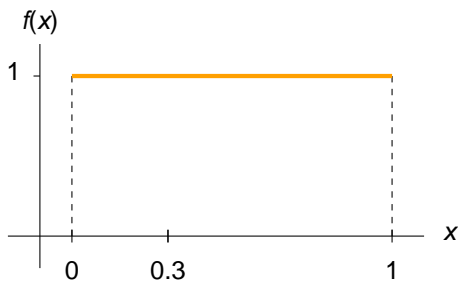
- The TI-83 will return a random number between 0 and 1 if we enter `rand` and press `ENTER`.
- These numbers have a uniform distribution from 0 to 1.
- Let X be the random number whose value is determined by the `rand` function.

Example



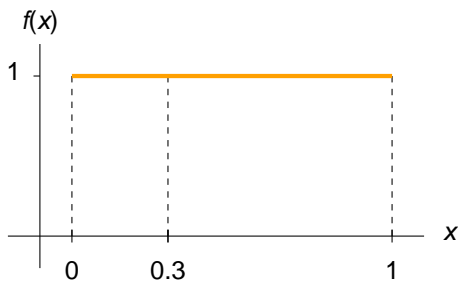
What is the probability that the random number is at least 0.3?

Example



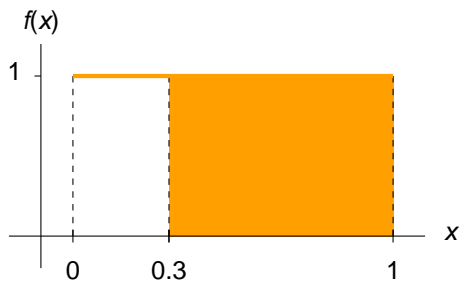
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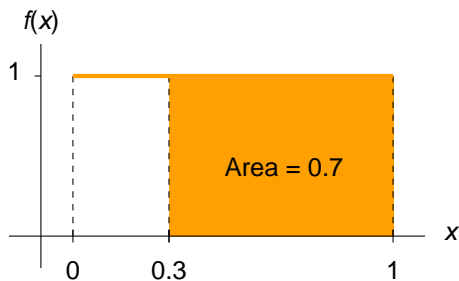
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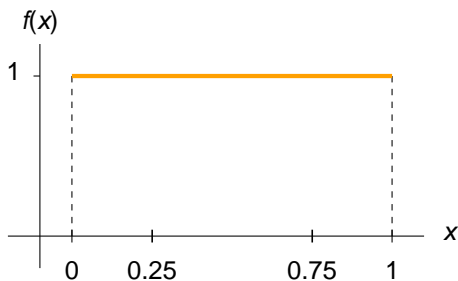
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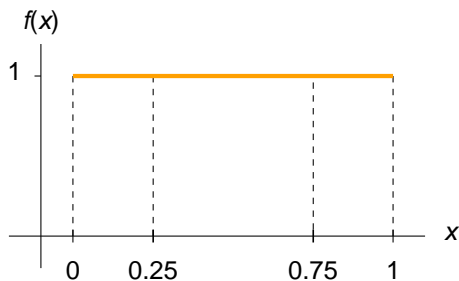
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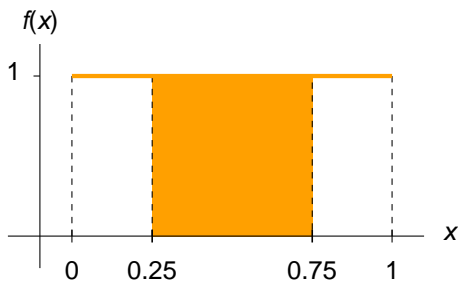
What is the probability that the random number is between 0.25 and 0.75?

Example



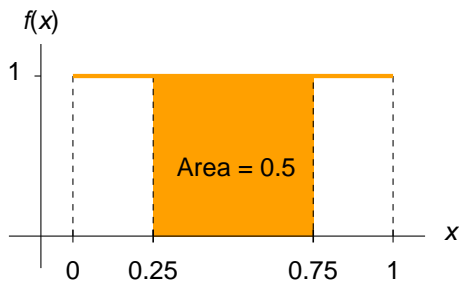
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What is the probability that the random number is between 0.25 and 0.75?

Example

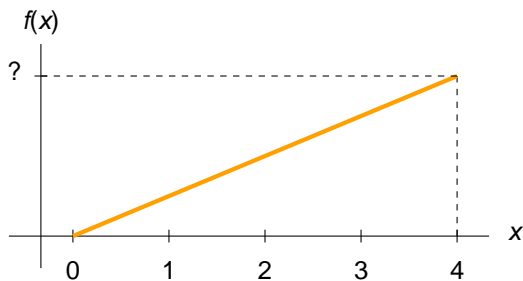


What is the probability that the random number is between 0.25 and 0.75?

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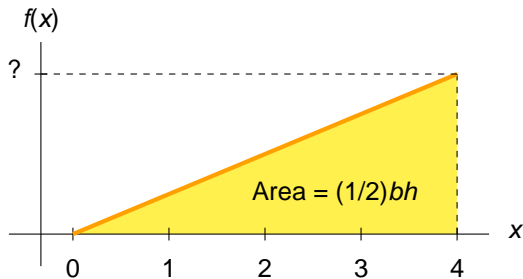
- 2 Assignment

A Non-Uniform Distribution



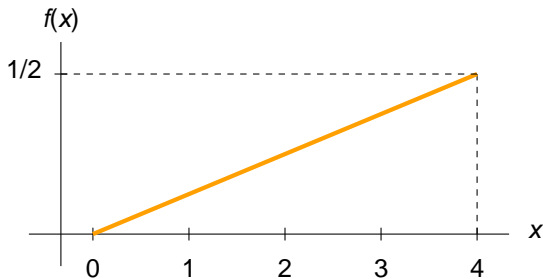
What is the height of this distribution?

A Non-Uniform Distribution



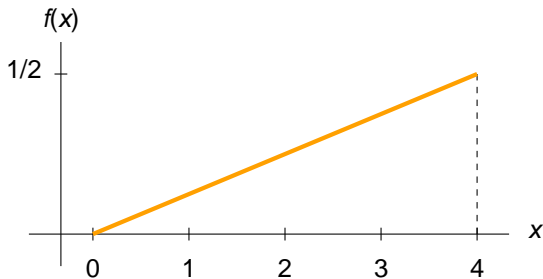
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A Non-Uniform Distribution



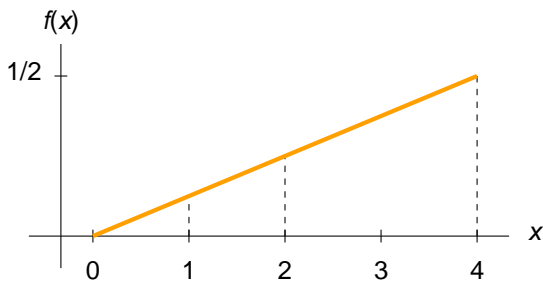
What is the height of this distribution?

A Non-Uniform Distribution



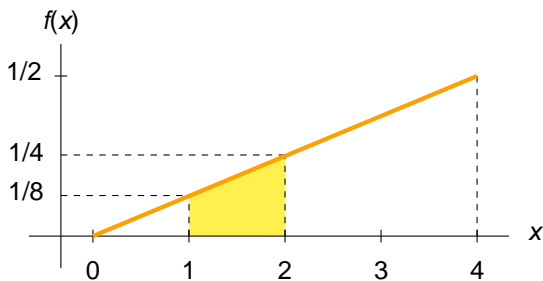
What is the probability that $1 \leq X \leq 2$?

A Non-Uniform Distribution



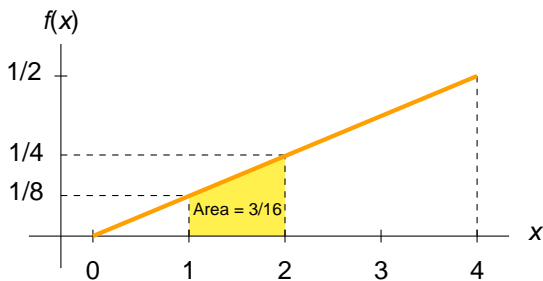
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A Non-Uniform Distribution



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A Non-Uniform Distribution



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Outline

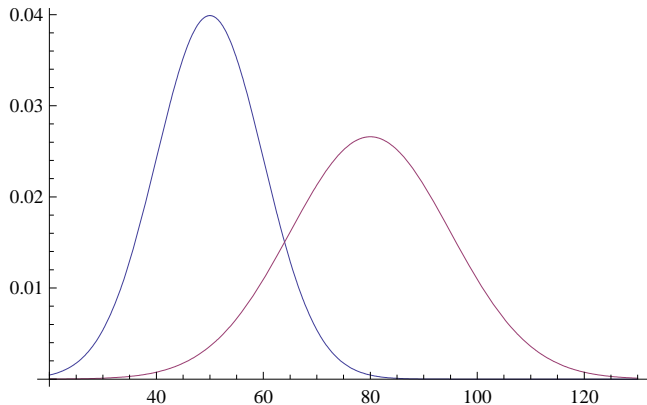
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Drug User vs. Non-drug User

- A subject is either a drug user or a non-drug user.
- To determine which, we measure the level of Substance “X” in his blood.
- If he is not a drug user, the level of X has the distribution $N(50, 10)$.
- If he is a drug user, the level of X has distribution $N(80, 15)$.

Drug User vs. Non-drug User



Drug User vs. Non-drug User

- We take a blood sample and measure the amount of X .
- Decision Rule: If the level is more than 65, then we will decide that he is a drug user.

Drug User vs. Non-drug User

- If he is not a drug user, what is the probability that the test will conclude that he is a drug user?

Drug User vs. Non-drug User

- If he is not a drug user, what is the probability that the test will conclude that he is a drug user?
- That would be a **false positive**.

Drug User vs. Non-drug User

- If he is not a drug user, what is the probability that the test will conclude that he is a drug user?
- That would be a **false positive**.
- If he is a drug user, what is the probability that the test will conclude that he is not a drug user?

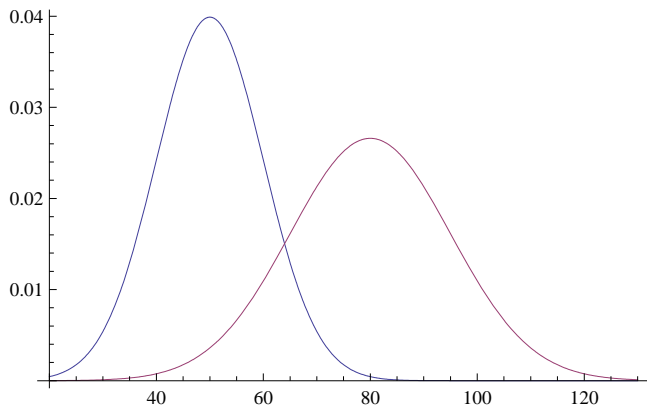
Drug User vs. Non-drug User

- If he is not a drug user, what is the probability that the test will conclude that he is a drug user?
- That would be a **false positive**.
- If he is a drug user, what is the probability that the test will conclude that he is not a drug user?
- That would be a **false negative**.

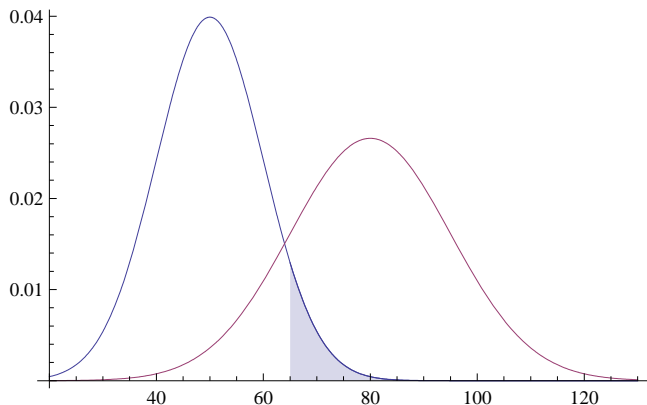
Drug User vs. Non-drug User

- If he is not a drug user, what is the probability that the test will conclude that he is a drug user?
- That would be a **false positive**.
- If he is a drug user, what is the probability that the test will conclude that he is not a drug user?
- That would be a **false negative**.
- Which type of error is more serious?

Drug User vs. Non-drug User

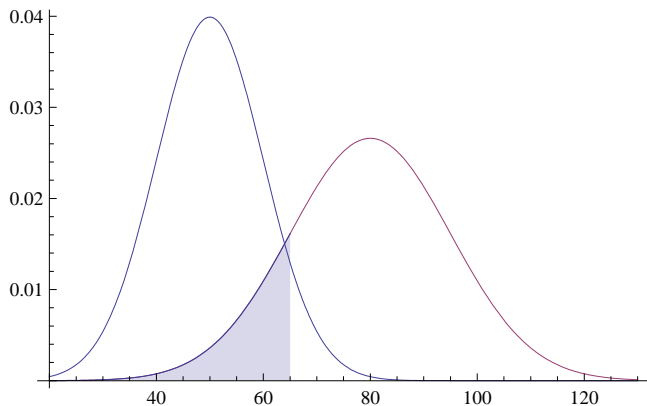


Drug User vs. Non-drug User



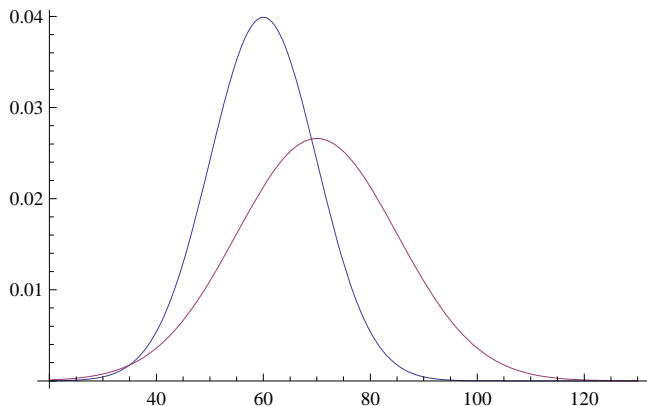
$$\text{normalcdf}(65, E99, 50, 10) = \mathbf{0.0668}$$

Drug User vs. Non-drug User



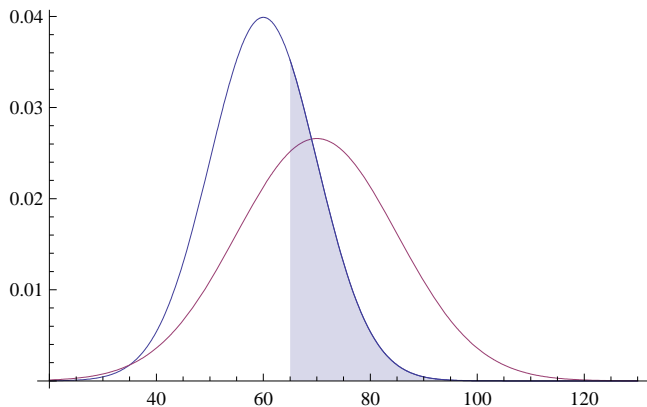
$$\text{normalcdf}(-E99, 65, 80, 15) = 0.1587$$

Drug User vs. Non-drug User



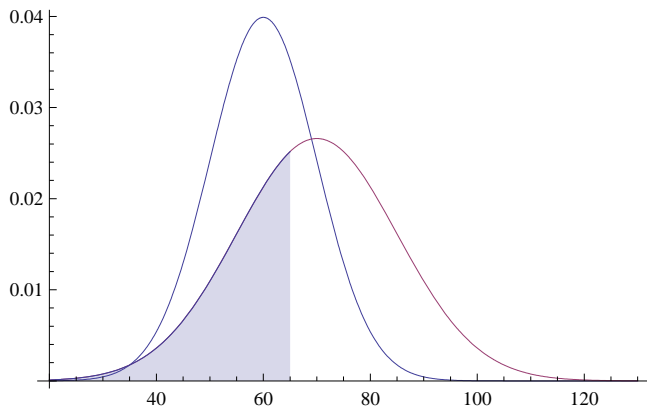
If the means are very close together ($\mu_1 = 60$, $\mu_2 = 70$), then the probabilities of false positives and false negatives will be large.

Drug User vs. Non-drug User



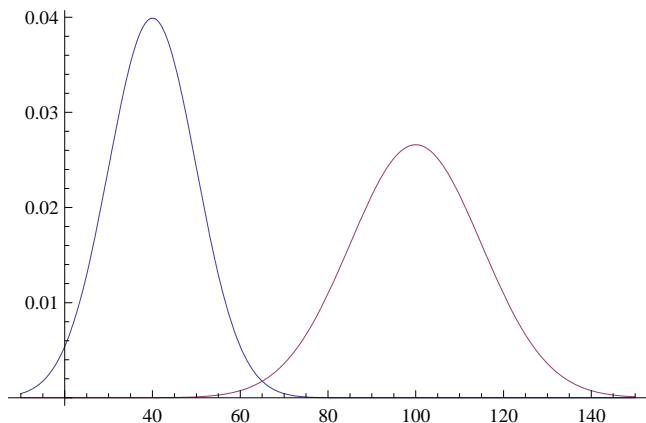
$$\text{normalcdf}(65, E99, 60, 10) = 0.3085$$

Drug User vs. Non-drug User



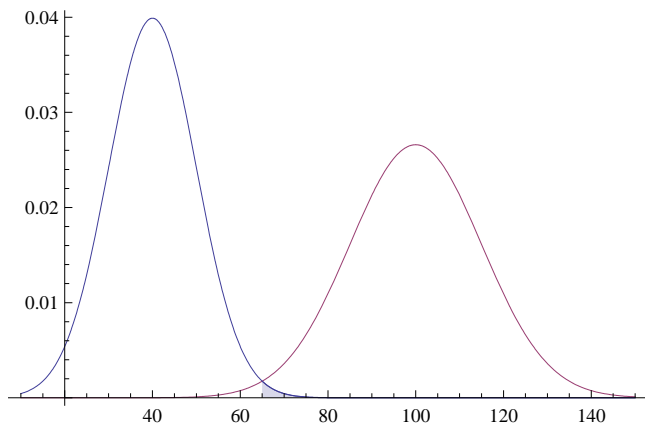
$$\text{normalcdf}(-E99, 65, 70, 15) = 0.3694$$

Drug User vs. Non-drug User



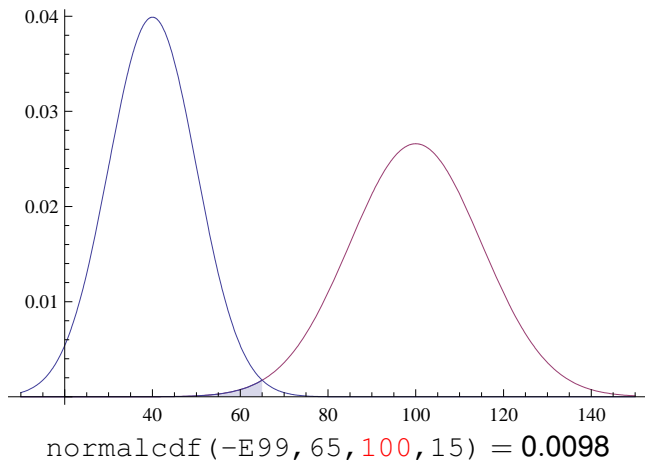
If the means are far apart ($\mu_1 = 40$, $\mu_2 = 100$), then the probabilities of false positives and false negatives will both be very small.

Drug User vs. Non-drug User



$$\text{normalcdf}(65, E99, 40, 10) = 0.0062$$

Drug User vs. Non-drug User



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Assignment

Assignment

- Read Sections 12.7.
- Apply Your Knowledge: 18, 19.
- Check Your Skills: 30, 31.
- Exercises 52, 54.