Confidence Intervals
Sections 16.1, 16.2
Lecture 30

Robb T. Koether

Hampden-Sydney College

Mon, Mar 14, 2016

Outline

Outline

Reasoning

- How do we use a sample mean $\overline{x}$ to estimate a population mean μ?
- The value $\overline{x}$ gives us a point estimate of μ, but provides no indication of how reliable that point estimate is.
- We would rather have an interval estimate, which is the point estimate, plus or minus a margin of error.

The Margin of Error

Fact

The distance from $\overline{x}$ to μ is the same as the distance from μ to $\overline{x}$.

- Therefore, if there is a 95% chance that $\overline{x}$ is within 3 units of μ, for example, then there is a 95% chance that μ is within 3 units of $\overline{x}$.
- The principle allows us to use either one of the formulations.

Outline

Example

Example (Test Scores)

- Suppose that (somehow) we know that a large set of test scores has a standard deviation of $\sigma = 12$, but we do not know the mean μ.
- We plan to take a sample of size $n = 100$ and compute the sample mean $\overline{x}$.
- What can we say about $\overline{x}$ before we compute it?

Example

Example (Test Scores)

- The sampling distribution of $\overline{x}$ is normal with mean μ and standard deviation
$$\frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{100}} = 1.2.$$
- What is the probability that $\overline{x}$ is within 2 standard deviations of μ?
- That is, that $\overline{x}$ is between $\mu - 2.4$ and $\mu + 2.4$?
- By the 68-95-99.7 Rule, the probability is 95%.

Example

Example (Test Scores)

- Because there is a 95% chance that $\overline{x}$ is within 2.4 points of μ...
- ...it follows that there is a 95% chance that μ is within 2.4 points of $\overline{x}$.

Example

Example (Test Scores)

- Now we take our sample and find that $\overline{x} = 82.5$.
- So there is a 95% chance that μ is between $\overline{x} - 2.4$ and $\overline{x} + 2.4$.
- That is, a 95% chance that $80.1 \leq \mu \leq 84.9$.

Example

Example (Test Scores)

- Now we take our sample and find that $\overline{x} = 82.5$.
- So there is a 95% chance that μ is between $\overline{x} - 2.4$ and $\overline{x} + 2.4$.
- That is, a 95% chance that $80.1 \leq \mu \leq 84.9$.
- Actually, we should say that we are 95% confident that μ lies between 80.1 and 84.9.

Outline

Summary

Summary

- Let the population have mean μ and standard deviation σ and let n be the sample size.

Summary

- Let the population have mean μ and standard deviation σ and let n be the sample size.
- Then $\overline{x}$, as a random variable, has a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.

Summary

Summary

- Let the population have mean μ and standard deviation σ and let n be the sample size.
- Then $\overline{x}$, as a random variable, has a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.
- Using the 68-95-99.7 Rule, we say that 95% of the intervals

$$\overline{x} \pm 2\left(\frac{\sigma}{\sqrt{n}}\right)$$

generated from simple random samples will contain μ and the other 5% will not contain μ.

Summary

Summary

- Let the population have mean μ and standard deviation σ and let n be the sample size.
- Then $\overline{x}$, as a random variable, has a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.
- Using the 68-95-99.7 Rule, we say that 95% of the intervals

$$\overline{x} \pm 2\left(\frac{\sigma}{\sqrt{n}}\right)$$

  generated from simple random samples will contain μ and the other 5% will not contain μ.
- When we generate a single such interval, we say that we are 95% confident that it contains μ.

Outline

Assignment

Assignment

- Read Sections 16.1, 16.2.
- Apply Your Knowledge: 1, 2, 4.