The Testing Procedure
Sections 17.3, 17.4
Lecture 33

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Outline

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1. Statistical Testing
   - Test Statistic
   - *p*-Value
   - The Significance Level
2. The Testing Procedure
3. Hypothesis Testing on the TI-83
4. Assignment

Statistical Testing Procedure

- We have outlined the reasoning behind statistical testing.
- Now we will develop a step-by-step procedure for carrying out the test.

Outline

Test Statistic

Definition (Test Statistic)

A test statistic is a statistic that measures how far the data diverge from what we would expect to see if the null hypothesis were true.

- Each type of statistical test will have its own specific test statistic.
- For tests concerning the mean μ, the test statistic will measure the number of standard deviations between the observed value and the expected value.

Example

Fact

The numbers turned up by a fair die have mean $\mu = 3.5$ *and standard deviation* $\sigma = 1.7078$*, in the long run.*

Example (Roll a Die)

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- Suppose the 100 rolls produces an average of $\overline{x} = 3.9$?
  - How many standard deviations is 4.1 from 3.9?
  - How likely is it that an observation would be at least that many standard deviations from 3.5 (in either direction)?
  - Does these data appear to be sufficiently unlikely (if H_0 were true) that we may reject the null hypothesis?

Test Statistic

- Based on the last example, we see that the test statistic in this situation is

$$\boxed{z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}}$$

where μ_0 is the hypothetical value of μ.

Outline

p-Value

Definition (p-Value)

The p-value of an observation x in a test of hypotheses is the probability that one would observe a value *at least as extreme as* x if the null hypothesis were true.

- The *smaller* the p-value, the less likely it is that we would observe such a value if H_0 were true.
- Therefore, the *smaller* the p-value, the greater our doubt that H_0 is true.
- And, therefore, the more likely we are to reject H_0.

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- Therefore, the *smaller* the p-value, the greater our doubt that H_0 is true.
- And, therefore, the more likely we are to reject H_0.
- But how small is small enough?

p-Value

Example (Rolling a Die)

- In the die-rolling example, the p-value was

$$\begin{aligned} p\text{-value} &= 2 \times P(z > 2.342) \\ &= 0.0192. \end{aligned}$$

p-Value

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- (Why “times 2?”)

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  - Simply present the reader with the p-value and let the reader draw his own conclusion.
  - Specify a level below which you will reject H_0 in favor of H_a.
- That level is called the level of significance and is denoted by the letter α.

The Significance Level

- Typically, $\alpha = 0.05$, but it may vary with the circumstances.
- The rule is
  - If the *p*-value $< \alpha$, then reject H_0.
  - If the *p*-value $> \alpha$, then do not reject H_0.
- In the die-rolling example, if $\alpha = 0.05$, then we would reject H_0 and conclude that the die is not fair.

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  - If the *p*-value $> \alpha$, then do not reject H_0.
- In the die-rolling example, if $\alpha = 0.05$, then we would reject H_0 and conclude that the die is not fair.
- What is α had been 0.01?

Outline

The Testing Procedure

- The procedure that we will follow is adapted from the one in the textbook on page 401.
- When you work a hypothesis-testing problem, it is required that you write out all of the steps.

The Testing Procedure

The Testing Procedure

(1) State the null and alternative hypotheses.

(2) If a significance level α is used, state its value.

(3) Give the formula for the test statistic.

(4) Calculate the value of the test statistic.

(5) Calculate the *p*-value.

(6) If a significance level is used, then draw a conclusion.

The Testing Procedure

(1) State the null and alternative hypotheses.
- The null hypothesis is of the form

$$H_0 : \textit{parameter} = \mu_0.$$

- The alternative hypothesis is of a similar form, except it uses either $<$, $>$, or $\neq$.

(2) If a significance level α is used, state its value.
- For example, $\alpha = 0.05$.

The Testing Procedure

The Testing Procedure

(3) Give the formula for the test statistic.

- We will have different formulas for different situations.
- Right now the only formula is

$$z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}.$$

(4) Calculate the value of the test statistic.

- Substitute the values of $\overline{x}$, μ_0, σ, and n.
- Calculate the value of z.

The Testing Procedure

(5) Calculate the *p*-value.

- If $H_a : \mu < \mu_0$, then *p*-value $= P(z <$ test statistic$)$.
- If $H_a : \mu > \mu_0$, then *p*-value $= P(z >$ test statistic$)$.
- If $H_a : \mu \neq \mu_0$, then *p*-value $= 2 \cdot P(z > |$test statistic$|)$.

(6) If a significance level is used, then draw a conclusion.

- If *p*-value $< \alpha$, then reject H_0.
- If *p*-value $> \alpha$, then do not reject H_0.

Example

Example (The Testing Procedure)

- A model of wood stove (similar to the one I own) advertises that it emits 1.1 grams of particles per hour, well below the current EPA limit of 4.5 grams/hour.
- Suppose I take 25 sample emissions and find an average of 1.29 g/hr and a standard deviation of 0.32 g/hr.
- Test at the 5% level of significance the hypothesis that the stove emits an average of 1.1 g/hr.

Example

Example (The Hypotheses)

- State the hypotheses.

$$H_0 : \mu = 1.1$$
$$H_a : \mu > 1.1$$

Example

Example (The Significance Level)

- State the significance level.

$$\alpha = 0.05.$$

Example

Example (The Test Statistic)

- Write the formula for the test statistic.

$$z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}$$

Example

Example (Calculate the Value of the Test Statistic)

- Calculate the value of the test statistic.

$$
\begin{aligned}
z &= \frac{1.29 - 1.1}{0.32/\sqrt{25}} \\
&= \frac{0.19}{0.064} \\
&= 2.969.
\end{aligned}
$$

Example

Example (Calculate the *p*-Value)

- Calculate the *p*-value.

$$\begin{aligned} p\text{-value} &= P(z > 2.969) \\ &= \texttt{normalcdf(2.969,E99)} \\ &= 0.0015. \end{aligned}$$

Example (Draw a Conclusion)

- Because p-value $< \alpha$, we reject H_0 and conclude that the average rate of particle emission is greater than 1.1 g/hr.

Outline

Hypothesis Testing on the TI-83

TI-83 Hypothesis Testing for the Mean

- Press `STAT`.
- Select `TESTS`.
- Select `Z-Test`.
- Press `ENTER`. A window appears requesting information.
- Select `Data` if you have the sample data entered into a list.
- Otherwise, select `Stats`.

TI-83 Hypothesis Testing for the Mean (Stats Option)

- Enter μ_0, the hypothetical mean.
- Enter σ. (Remember, σ is known.)
- Enter $\overline{x}$.
- Enter n, the sample size.
- Select the type of alternative hypothesis.
- Select `Calculate` and press `ENTER`.

TI-83 Hypothesis Testing for the Mean

- A window appears with the following information.
  - The title Z-Test.
  - The alternative hypothesis.
  - The value of the test statistic Z.
  - The p-value of the test.
  - The sample mean.
  - The sample size.

Outline

Assignment

Assignment

- Read Sections 17.3, 17.4.
- Apply Your Knowledge: 8, 9, 13, 15.
- Check Your Skills: 21, 22, 23, 25, 26.
- Exercises 30, 31, 32. Show all steps.
- Exercises 33, 38.