

Using the t Statistic

Sections 20.3, 20.4, 20.5

Lecture 36

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Outline

- 1 The Standard Error
- 2 Confidence Intervals Involving t
- 3 Hypothesis Testing Involving t
- 4 Assignment

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The Standard Error

Definition (Standard Error)

The **standard error** of \bar{x} is

$$SE = \frac{s}{\sqrt{n}}.$$

Example

Example (Estimating Wood Stove Emissions)

- Suppose I collect a sample of 9 measurements (g/hr) of the emissions from my wood stove.
- My measurements are

1.25, 0.85, 0.44, 1.49, 1.35, 1.50, 0.86, 1.17, 1.52

- Find \bar{x} and s .
- Compute a 95% confidence interval for μ .

Example

Example (Estimating Wood Stove Emissions)

- We find $\bar{x} = 1.159$ and $s = 0.3713$.
- To find the value of t^* , use the `invT()` function in the TI-84.

$$\text{invT}(0.025, 8) = -2.306004133$$

- Unfortunately, the TI-83 does not have the `invT()` function.

Example

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- Unfortunately, the TI-83 does not have the `invT()` function.
- However, I can program it into the TI-83 if you want.

Example

Example (Estimating Wood Stove Emissions)

- The 95% confidence interval is

$$\begin{aligned}\bar{x} \pm t^* \left(\frac{s}{\sqrt{n}} \right) \\ &= 1.159 \pm (2.306) \left(\frac{0.3713}{\sqrt{9}} \right) \\ &= 1.159 \pm 0.285.\end{aligned}$$

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Confidence Intervals Involving t

- The formula for a confidence interval for μ involving z is

$$\mu \pm z^* \left(\frac{\sigma}{\sqrt{n}} \right).$$

- We follow the same pattern when using t :

$$\mu \pm t^* \left(\frac{s}{\sqrt{n}} \right).$$

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Hypothesis Testing Involving t

- CCC

Example

Example (Testing Wood Stove Emissions)

- Use the wood stove emissions data to test whether the stove is emitting more than 1.1 g/hr of particles.
- My measurements are

1.25, 0.85, 0.44, 1.49, 1.35, 1.50, 0.86, 1.17, 1.52

- Test the hypothesis $H_0 : \mu = 1.1$ against the hypothesis $H_a : \mu > 1.1$ at the 0.01 level of significance.

Example

Example (The t Test – Hypotheses)

- The hypotheses are

$$H_0 : \mu = 1.1$$

$$H_a : \mu > 1.1$$

Example

Example (The t Test – Level of Significance)

- The level of significance is

$$\alpha = 0.01.$$

Example

Example (The t Test – Test Statistic)

- The test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Example

Example (The t Test – Value of the Test Statistic)

- The value of the test statistic is

$$\begin{aligned}t &= \frac{1.159 - 1.1}{0.3713/\sqrt{9}} \\ &= \frac{0.059}{0.124} && = 0.4758.\end{aligned}$$

Example

Example (The t Test – p -Value)

- The p -value is

$$\begin{aligned} p\text{-value} &= P(t > 0.4758) \\ &= 0.3235. \end{aligned}$$

Example

Example (The t Test – Conclusion)

- The conclusion is not to reject H_0 .
- We do not have sufficient evidence to prove statistically that the emissions exceed 1.1 g/hr.

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Assignment

Assignment

- Read Sections 20.3, 20.4, 20.5.
- Apply Your Knowledge: 5, 8, 9.
- Check Your Skills: 20, 21, 23, 24.
- Exercises 28, 29, 30, 37, 46.