Confidence Intervals for Proportions
Sections 22.2, 22.3
Lecture 41

Robb T. Koether

Hampden-Sydney College

Mon, Apr 4, 2016

Outline

Outline

Confidence Intervals for p

- The standard deviation of $\hat{p}$ is $\sqrt{\dfrac{p(1-p)}{n}}$, but in practice we do not know p.

Confidence Intervals for p

- The standard deviation of $\hat{p}$ is $\sqrt{\dfrac{p(1-p)}{n}}$, but in practice we do not know p.
- We use $\hat{p}$ as an estimator of p.

Confidence Intervals for p

- The standard deviation of $\hat{p}$ is $\sqrt{\dfrac{p(1-p)}{n}}$, but in practice we do not know p.
- We use $\hat{p}$ as an estimator of p.
- This gives us the standard error of $\hat{p}$:

$$\text{SE}_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

Confidence Intervals for p

- Confidence intervals for p follow the usual pattern:

$$\text{(point estimate)} \pm \text{(margin of error)}.$$

Confidence Intervals for p

- Confidence intervals for p follow the usual pattern:

$$\text{(point estimate)} \pm \text{(margin of error)}.$$

- The margin of error is

$$\text{margin of error} = z^*\text{(standard error)}.$$

Confidence Intervals for p

- Confidence intervals for p follow the usual pattern:

$$\text{(point estimate)} \pm \text{(margin of error)}.$$

- The margin of error is

$$\text{margin of error} = z^*(\text{standard error}).$$

- The standard error is

$$\text{standard error} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

Confidence Intervals for p

- Thus, the confidence interval is

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

Confidence Intervals for p

- The normality assumption (use of z^*) is justified provided the sample contains
  - At least 15 members with the property, and
  - At least 15 members without the property.

Example

Example (Confidence Intervals for p)

- A recent Quinnipiac poll found that 43% of 834 Republicans surveyed support Trump.

Example

Example (Confidence Intervals for p)

- A recent Quinnipiac poll found that 43% of 834 Republicans surveyed support Trump.
- Find a 95% confidence interval for the true proportion who support Trump.

Example

Example (Confidence Intervals for p)

- We have $n = 834$ and $\hat{p} = 0.43$.

Example

Example (Confidence Intervals for p)

- We have $n = 834$ and $\hat{p} = 0.43$.
- The standard error is

Example

Example (Confidence Intervals for p)

- We have $n = 834$ and $\hat{p} = 0.43$.
- The standard error is

$$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.43)(0.57)}{843}}$$

Example

Example (Confidence Intervals for p)

- We have $n = 834$ and $\hat{p} = 0.43$.
- The standard error is

$$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.43)(0.57)}{843}}$$
$$= \sqrt{0.00029388}$$

Example

Example (Confidence Intervals for p)

- We have $n = 834$ and $\hat{p} = 0.43$.
- The standard error is

$$\begin{aligned}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} &= \sqrt{\frac{(0.43)(0.57)}{843}} \\ &= \sqrt{0.00029388} \\ &= 0.017.\end{aligned}$$

Example

Example (Confidence Intervals for p)

- The confidence interval is

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.43 \pm (1.960)(0.017)$$
$$= 0.43 \pm 0.0336.$$

Example

Example (Confidence Intervals for p)

- Recompute the confidence for $n = 100$. What is the effect?

Example

Example (Confidence Intervals for p)

- Recompute the confidence for $n = 100$. What is the effect?
- Recompute the confidence for $n = 3000$. What is the effect?

Example

Example (Confidence Intervals for p)

- Recompute the confidence for $n = 100$. What is the effect?
- Recompute the confidence for $n = 3000$. What is the effect?
- Recompute the confidence using $\hat{p} = 0.20$. What is the effect?

Example

Example (Confidence Intervals for p)

- Recompute the confidence for $n = 100$. What is the effect?
- Recompute the confidence for $n = 3000$. What is the effect?
- Recompute the confidence using $\hat{p} = 0.20$. What is the effect?
- Recompute the confidence using $\hat{p} = 0.10$. What is the effect?

Outline

Determining the Sample Size

- We just saw that the margin of error depends on
  - The population proportion, and
  - The level of confidence, and
  - The sample size.
- How can we achieve a specific margin of error, say 1%?

Determining the Sample Size

- We cannot change the population proportion.
- We do not want to lower the level of confidence.
- The only thing left is to increase the sample size.
- How large should it be?

Determining the Sample Size

- The formula for the margin of error is

$$\text{margin of error} = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

- Let m be the margin of error and solve the equation for n.
- We get:

$$n = \left(\frac{z^*}{m}\right)^2 \hat{p}(1-\hat{p}).$$

Example

Example

Determining the Sample Size

- Suppose Donald Trump has the support of about 42% of registered Republicans.
- How large must a sample be in order for the margin of error of a 95% confidence interval to be 3%?
- For a margin of error of 1%?
- What if we did not assume that $p = 0.42$?

Outline

Assignment

Assignment

- Read Section 22.1, 22.2.
- Apply Your Knowledge: 1, 2, 3, 4.
- Check Your Skills: 15, 16, 17, 18.
- Exercises 26, 27, 30, 31, 33.