

# Hypothesis Tests Concerning Two Proportions

## Section 23.5

### Lecture 44

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- 1 Hypothesis Tests Concerning Two Proportions
- 2 Assignment

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2 Assignment

# Hypothesis Tests Concerning Two Proportions

- Our procedure will follow the same 6 steps as always.
  1. State the hypotheses.
  2. Give the value of  $\alpha$ .
  3. Write the formula for the test statistic.
  4. Calculate the value of the test statistic.
  5. Calculate the  $p$ -value.
  6. Draw a conclusion.

# Hypothesis Tests Concerning Two Proportions

1. The null hypothesis is

$$H_0: p_1 = p_2$$

The alternative hypothesis is one of

$$H_a: p_1 \neq p_2$$

$$H_a: p_1 < p_2$$

$$H_a: p_1 > p_2$$

2. State the value of  $\alpha$ .

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- Thus, we get a better estimate of that common proportion if we **pool** the data.



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- Under the assumption that  $p_1 = p_2$ , it follows that  $\hat{p}_1$  and  $\hat{p}_2$  are both estimators of the common population proportion.
- Thus, we get a better estimate of that common proportion if we **pool** the data.
- That is, put the two samples together into one sample and compute a single  $\hat{p}$ .

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- The total number of successes is  $x_1 + x_2$  out of a total of  $n_1 + n_2$ .

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- The number of “successes” in the second sample is  $x_2 = \hat{p}_2 n_2$ .
- The total number of successes is  $x_1 + x_2$  out of a total of  $n_1 + n_2$ .
- The **pooled estimate**  $\hat{p}$  is

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}.$$

# Pooling the Data

- Then the standard error becomes

$$\begin{aligned} SE_{\hat{p}_1 - \hat{p}_2} &= \sqrt{\frac{\hat{p}(1 - \hat{p})}{n_1} + \frac{\hat{p}(1 - \hat{p})}{n_1}} \\ &= \sqrt{\hat{p}(1 - \hat{p}) \left( \frac{1}{n_1} + \frac{1}{n_1} \right)}. \end{aligned}$$

# Hypothesis Tests Concerning Two Proportions

3. The formula for the test statistic is

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Calculate  $\hat{p}$ .

4. Then substitute the values of  $\hat{p}$ ,  $n_1$ , and  $n_2$  to calculate the value of  $z$ .
5. Use `normalcdf` to find the  $p$ -value.
6. Draw the conclusion.

# Example

## Example

- A survey conducted by Rasmussen reports that 80% of Republicans believe that illegal aliens should not be allowed to vote, while 52% of Democrats believe that they should not be allowed to vote.
- Assume that the survey included 350 Republicans and 400 Democrats.
- Test for the difference in proportions using  $\alpha = 0.01$ .



# Example

## Example

- One year ago, 42% of Democrats in a survey opposed allowing illegal aliens to vote. Today, in a survey, 52% oppose it.
- Assume that both surveys included 400 Democrats.
- Test for the difference in proportions using  $\alpha = 0.01$ .

1 Hypothesis Tests Concerning Two Proportions

2 Assignment

# Assignment

## Assignment

- Read Section 23.5.
- Apply Your Knowledge: 4, 5, 6.
- Check Your Skills: 11, 12, 14.
- Exercises 20, 21, 22, 23, 26.