$(31, 58, 73, 90, 97), \overline{x} = 71.3.$

1. (10 pts) $(0, 2, 7, 7, 10)$, $\overline{x} = 4.8$.

   This is *not* a hypothesis-testing problem. It asks for a probability concerning $\hat{p}$. According to the Central Limit Theorem, the pdf of $\hat{p}$ is normal with mean 0.60 and standard deviation $\sqrt{\frac{(0.60)(0.40)}{100}} = 0.04899$. So find `normalcdf(-E99,0.50,0.60,0.04899)`, which is 0.02061.

   A number of people worked it as a hypothesis-testing problem. You can get the right answer that way, because the probability you need to find turns out to be the p-value when testing the hypothesis $H_0 : p = 0.60$ against $H_1 : p < 0.60$ and the sample proportion is $\hat{p} = 0.50$. However, if you choose to work the problem this way, I would expect you to clearly identify 0.02061 as the answer.

2. (15 pts) $(2, 9, 12, 13, 15)$, $\overline{x} = 10.7$.

   The seven steps of the hypothesis test are

   Step 1: $H_0 : p = 0.80$ vs. $H_1 : p < 0.80$

   Step 2: $\alpha = 0.05$

   Step 3: We have one sample and we are measuring the proportion, so the test statistic is $Z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$.

   Step 4: Compute $\hat{p} = \frac{423}{558} = 0.7581$. Then evaluate the formula:

   $$Z = \frac{0.7581 - 0.80}{\sqrt{\frac{(0.80)(0.20)}{558}}} = \frac{-0.0419}{0.01693} = -2.475.$$

   Step 5: The test is one-tailed to the left, so compute `normalcdf(-E99,-2.475,0.80,0.01693)` and get p-value $= 0.006662$.

   Step 6: Because the p-value is less than α, the decision is to reject H_0.

   Step 7: The conclusion is that less than 80% of Virginians own guns.

   You can get the answers to steps 4 and 5 by using `1-PropZTest`. Enter $p_0 = 0.80$, $x = 423$, $n = 558$, and choose $< p_0$ for the alternative. It will report $z = -2.4765$ and $p = .006634$.

3. (15 pts) $(0, 15, 15, 15, 15)$, $\overline{x} = 12.3$.

   We have the statistic $\hat{p} = 0.7581$. Use the formula $\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. You should get

   $$0.7581 \pm 1.96\sqrt{\frac{(0.7581)(0.2419)}{558}} = 0.7581 \pm 0.03553.$$

You could also get the answer by using `1-PropZInt`. Let $x = 423$, $n = 558$, and the confidence level be 0.95. The calculator gives the interval $(0.72253, 0.7936)$, which is the same interval as the one above.

4. (15 pts) $(6, 11, 13, 15, 15)$, $\overline{x} = 12.5$.

   In this problem, we are comparing the means of two samples to estimate the difference between the population means μ_1 and μ_2. The seven steps of the hypothesis test are

   Step 1: $H_0 : \mu_1 = \mu_2$ vs. $H_1 : \mu_1 > \mu_2$

   Step 2: $\alpha = 0.05$

   Step 3: The test statistic is $Z = \dfrac{\overline{x}_1 - \overline{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$, where $s_p = \sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$.

   Step 4: Use these formulas to compute

   $$s_p = \sqrt{\frac{(6999)0.13^2 + (6999)0.11^2}{13998}} = 0.1204$$

   and then

   $$Z = \frac{2.20 - 2.18}{0.1204\sqrt{\frac{1}{7000} + \frac{1}{7000}}} = \frac{0.02}{0.002035} = 9.826.$$

   Step 5: The p-value is `normalcdf(9.826,E99)` $= 4.432 \times 10^{-23}$.

   Step 6: Reject H_0.

   Step 7: Conclude that the average price of gas on Oct. 20 was higher than it was on Nov. 3.

   You could also get the answers to steps 4 and 5 by using either `2-SampZTest` or `2-SampTTest`. Because the sample sizes are so large, it does not matter which you use. If you use `2-SampZTest`, then let $\sigma_1 = 0.13$, $\sigma_2 = 0.11$, $\overline{x}_1 = 2.20$, $n_1 = 7000$, $\overline{x}_2 = 2.18$, and $n_2 = 7000$. Let the alternative be $> \mu_2$. The calculator reports that $z = 9.826$ and $p = 4.429 \times 10^{-23}$.

   If you use `2-SampTTest`, then let $\overline{x}_1 = 2.20$, $s_1 = 0.13$, $n_1 = 7000$, $\overline{x}_2 = 2.18$, $s_2 = 0.11$, and $n_2 = 7000$. Let the alternative be $> \mu_2$ and say "Yes" to pooling. The calculator reports that $t = 9.826$ and $p = 5.148 \times 10^{-23}$.

5. (15 pts) $(2, 10, 12, 15, 15)$, $\overline{x} = 11.7$.

   In this problem, we are comparing sample proportions from two samples. The seven steps of the hypothesis test are

   Step 1: The researchers are interested in any difference at all, so the hypotheses are $H_0 : p_1 = p_2$ vs. $H_1 : p_1 \neq p_2$

   Step 2: $\alpha = 0.01$

Step 3: The test statistic is $Z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$, where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$.

Step 4: Compute $\hat{p}_1 = \frac{18}{200} = 0.09$, $\hat{p}_2 = \frac{14}{250} = 0.056$, and $\hat{p} = \frac{18+14}{200+250} = \frac{32}{450} = 0.07111$. Then compute

$$Z = \frac{0.09 - 0.056}{\sqrt{(0.07111)(0.92889)\left(\frac{1}{200} + \frac{1}{250}\right)}} = \frac{0.034}{0.02348} = 1.394.$$

Step 5: The p-value is $2 \times$ `normalcdf(1.394,E99)` $= 0.1633$. We multiply by 2 because the test is a two-tailed test.

Step 6: Accept H_0.

Step 7: The conclusion is that there is no difference in the rate of defective items produced by the two processes.

This problem can also be worked using `2-PropZTest`. Let $x_1 = 18$, $n_1 = 200$, $x_2 = 14$, and $n_2 = 250$, and let the alternative be $\neq p_2$. The result is $z = 1.394$ and $p = 0.1632$.

6. (15 pts) $(2, 8, 11, 13, 15)$, $\overline{x} = 10.0$.

This is a hypothesis test concerning μ and involving one sample. The seven steps are

Step 1: $H_0 : \mu = 47$ vs. $H_1 : \mu > 47$

Step 2: $\alpha = 0.05$

Step 3: The sample is small, we are using s instead of σ, and the population that we are sampling from is normal. Therefore, we must use the t distribution. The test statistic is $t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$.

Step 4: Enter the data into list $\texttt{L}_1$: `{47,61,49,47,47,63,45,48,48,45}`$\to\texttt{L}_1$. Use `1-Var Stats` to find the mean and standard deviation: $\overline{x} = 50$ and $s = 6.464$. Then compute

$$t = \frac{50 - 47}{6.464/\sqrt{10}} = \frac{3}{2.044} = 1.468.$$

Step 5: The p-value is `tcdf(1.468,E99,9)` $= 0.08808$.

Step 6: Accept H_0.

Step 7: Conclude that the average carbohydrate content is 47 g.

Use can also work this problem by using `T-Test`. Select `Data`. Then enter $\mu_0 = 47$, $\texttt{L}_1$ for the list, and $> \mu_0$ for the alternative. The calculator reports that $t = 1.468$ and $p = 0.08812$.

7. (15 pts) $(0, 0, 15, 15, 15)$, $\overline{x} = 8.6$.

   Since the situation is the same as in the previous problem, we will continue to use the t distribution. The formula is $\overline{x} \pm t\left(\frac{s}{\sqrt{n}}\right)$. Use the t table to find the value of t. We have 9 degrees of freedom and the confidence level is 0.90. So according to the table, $t = 1.833$. Compute

$$50 \pm 1.833\left(\frac{6.464}{\sqrt{10}}\right) = 50 \pm 3.767.$$

   You can also use `T-Interval`. Select `Data` and let the list be L_1 and the confidence level be 0.90. The result is the interval $(46.253, 53.747)$.