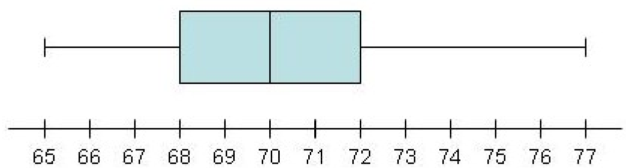
Math 121 **Test #2 Solutions** **Spring 2005**

1. (a) (Oops! The correct formula is $r = 1 + \left(\frac{p}{100}\right)(n-1)$. Sorry.) According to given formula, the minimum is 65 and the maximum is 77. For the first quartile, use $p = 25$ and $n = 25$ to compute the rank 6. Thus, Q1= the 6^{th} number, which is 68. Similarly, for the median, use $p = 50$ and get the number 70 and for Q3, use $p = 75$ and get the number 72.

   (b) $\text{IQR} = \text{Q3} - \text{Q1} = 72 - 68 = 4$.

   (c) The boxplot is



   (d) It appears to be very close to symmetric except for the right whisker, which suggests some skewness to the right.

   (e) There are no outliers. The $1.5 \times \text{IQR} = 6$, so all values are within the inner fences.

2. (a) The average is 19.

   (b) The standard deviation is 2.366. If you computed it by hand, you should have found $\sum (x - \overline{x})^2 = 28$ and $s^2 = \frac{28}{5} = 5.6$. Then $s = \sqrt{5.6} = 2.366$.

3. (a) Look up 1.56 in the normal table. The entry is 0.9406. Or use `normal(-E99,1.56)` and get 0.9406.

   (b) Subtract the answer in (a) from 1: $1 - 0.9406 = 0.0594$.

   (c) Look up -1.56 and get 0.0594. Then subtract $0.9406 - 0.0594 = 0.8812$.

   (d) Look up 0.7500 in the "area part" of the table. You have to choose between 0.7486 or 0.7517. These numbers give $z = 0.67$ or $z = 0.68$ as the 75^{th} percentile. Or use `invNorm(.75)` to get $z = 0.6745$.

4. (a) Compute the z-scores of 25 and 30: $z = \frac{25-28.5}{4.2} = -0.83$ and $z = \frac{30-28.5}{4.2} = 0.36$, respectively. Look these numbers up in the normal table and get 0.2033 and 0.6406. Subtract to get the answer 0.4373. Or you can use `normalcdf(-.83,.36)` or `normalcdf(25,30,28.5,4.2)` to get the answer.

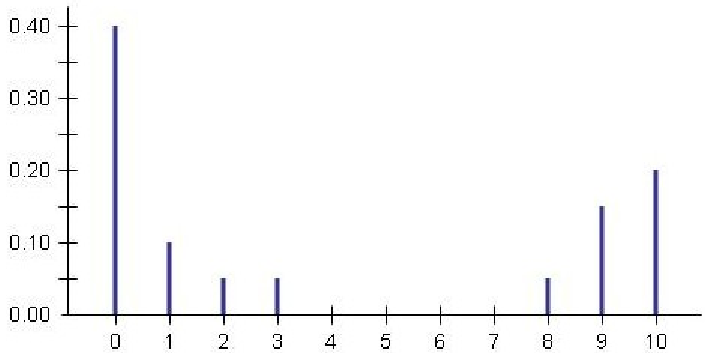
   (b) As in the previous problem, the 75^{th} percentile has a z-score of 0.67. Thus, the 75^{th} percentile in this case is $28.5 + (0.67)(4.2) = 31.314$.

5. (a) The direction of extreme is to the right, so α is the area to the right of 50 in the first distribution, $N(35, 10)$. Under H_0, the z-score of 50 is 1.5, so look up 1.5 in the normal table and get the area of 0.0668 to the right.

(b) β is the area to the left of 50 in the second distribution, $N(55, 10)$. The z-score is -0.5, so the area turns out to be 0.3085.

(c) Under H_0, the z-score of 45 is 1. So the area to the right of 45 is 0.1587.

6. (a) The stick graph.



(b) Add up the probabilities for 6, 7, 8, 9, and 10:

$$P(6, 7, 8, 9, \text{ or } 10) = 0.0 + 0.0 + 0.05 + 0.15 + 0.20 = 0.40.$$

(c) Compute

$$\begin{aligned} E(x) &= \sum x \cdot P(X = x) \\ &= (0)(0.40) + (1)(0.10) + (2)(0.05) + (3)(0.05) \\ &\quad +(4)(0.0) + (5)(0.0) + (6)(0.0) + (7)(0.0) \\ &\quad +(8)(0.05) + (9)(0.15) + (10)(0.20) \\ &= 4.1 \end{aligned}$$

(d) No. The actual scores are whole numbers and 4.1 is not a whole number.

7. (a) True. $\hat{p}$ is an unbiased estimator of p.

(b) True. The variability of $\hat{p}$ decreases as the sample size increases because of the $\sqrt{n}$ in the denominator of the formula $\sigma_{\hat{p}} = \frac{\sigma}{\sqrt{n}}$.

(c) False. If the sample size is small enough that either $np < 5$ or $n(1-p) < 5$, then the distribution of $\hat{p}$ is not normal.

8. (a) The sampling distribution of $\hat{p}$ is (approximately) normal with mean $\mu_{\hat{p}} = 0.60$ and standard deviation

$$\sigma_{\hat{p}} = \sqrt{\frac{(0.60)(0.40)}{24}} = 0.10.$$

(b) Yes, it is because $np = (24)(0.60) = 14.4 \geq 5$ and $n(1-p) = (24)(0.40) = 9.6 \geq 5$.

(c)

$$P(\hat{p} < 0.50) = P\left(z < \frac{0.50 - 0.60}{.10}\right) = P(Z < -1) = 0.1587.$$