Math 121 Chapter 14 Solutions Spring 2012

1. (10 pts)

(a) Multiply 0.25 by the sample size of 1120 to get 280 for each category.

	Class	Freshman	Sophomore	Junior	Senior
Enrollment	Actual	314	305	261	240
	Expected	(280)	(280)	(280)	(280)

(b) The seven steps:

1. H_0 : The populations are homogeneous.
H_1 : The populations are not homogeneous.

2. $\alpha = 0.05$.

3. $\chi^2 = \sum_{\text{all cells}} \frac{(O-E)^2}{E}$.

4.

$$\begin{aligned}\chi^2 &= \frac{(314-280)^2}{280} + \frac{(305-280)^2}{280} + \frac{(261-280)^2}{280} + \frac{(240-280)^2}{280} \\ &= 4.1286 + 2.2321 + 1.2893 + 5.7143 \\ &= 13.3643.\end{aligned}$$

5. p-value $= \chi^2$`cdf(13.3643,E99,3)` $= 0.0039$.

6. Reject H_0.

7. The populations are not homogeneous.

You could enter the observed counts into list $\mathtt{L_1}$ and the expected counts into list $\mathtt{L_2}$. Then compute $\mathtt{(L_1\text{-}L_2)^2/L_2}$. Then use `sum(Ans)` to sum up the values to get 13.36.

2. (10 pts) The seven steps:

1. $H_0 : p_1 = p_2 = \cdots = p_6 = \frac{1}{6}$.
$H_1 : H_0$ is not true.

2. $\alpha = 0.01$.

3. $\chi^2 = \sum_{\text{all cells}} \frac{(O-E)^2}{E}$.

4. The observed and expected counts are

	1	2	3	4	5	6
Observed	10	14	16	25	23	32
Expected	(20)	(20)	(20)	(20)	(20)	(20)

$$\begin{aligned}\chi^2 &= \frac{(10-20)^2}{20} + \frac{(14-20)^2}{20} + \frac{(16-20)^2}{20} + \frac{(25-20)^2}{20} + \frac{(23-20)^2}{20} + \frac{(32-20)^2}{20} \\ &= 5.0 + 1.8 + 0.8 + 1.25 + 0.45 + 7.2 \\ &= 16.5.\end{aligned}$$

5. p-value $= \chi^2$`cdf(16.5,E99,5)` $= 0.00555$.

6. Reject H_0.

7. The numbers do not all have probability $\frac{1}{6}$.

You could enter the observed counts into list $\texttt{L}_1$ and the expected counts into list $\texttt{L}_2$. Then compute $\texttt{(L}_1\texttt{-L}_2\texttt{)}^2\texttt{/L}_2$. Then use `sum(Ans)` to sum up the values to get 16.5.

3. (10 pts) The seven steps:

1. $H_0 : p_1 = p_2 = p_3 = 0.10, p_4 = p_5 = 0.20, p_6 = 0.30$.
$H_1 : H_0$ is not true.

2. $\alpha = 0.01$.

3. $\chi^2 = \displaystyle\sum_{\text{all cells}} \frac{(O - E)^2}{E}$.

4. The observed and expected counts are

	1	2	3	4	5	6
Observed	8	11	14	29	15	23
Expected	(10)	(10)	(10)	(20)	(20)	(30)

$$\begin{aligned}\chi^2 &= \frac{(8-10)^2}{10} + \frac{(11-10)^2}{10} + \frac{(14-10)^2}{10} + \frac{(29-20)^2}{20} + \frac{(15-20)^2}{20} + \frac{(23-30)^2}{30} \\ &= 0.4 + 0.1 + 1.6 + 4.05 + 1.25 + 1.6333 \\ &= 9.033.\end{aligned}$$

5. p-value $= \chi^2$`cdf(9.033,E99,5)` $= 0.1077$.

6. Accept H_0.

7. The probabilities stated in the null hypothesis are correct.

You could enter the observed counts into list $\texttt{L}_1$ and the expected counts into list $\texttt{L}_2$. Then compute $\texttt{(L}_1\texttt{-L}_2\texttt{)}^2\texttt{/L}_2$. Then use `sum(Ans)` to sum up the values to get 16.5.

4. (12 pts) You need to enter the data into a 2×10 matrix and then run the χ^2 test.

(a) (10 pts) The question says to test "that the two distributions are the same." That means to test whether they are *homogeneous*. The seven steps:

1. H_0 : The populations are homogeneous.
H_1 : The populations are not homogeneous.

2. $\alpha = 0.05$.

3. $\chi^2 = \displaystyle\sum_{\text{all cells}} \frac{(O - E)^2}{E}$.

4. $\chi^2 = 94.91$.

5. p-value $= \chi^2$`cdf(94.91,E99,9)` $= 1.678 \times 10^{-16}$.

6. Reject H_0.

7. The populations are not homogeneous.

(b) (2 pts) The expected number of incidents of crime in Ashland in 2007 is 299.8.

5. (12 pts) You need to enter the data into a 2×3 matrix and then run the χ^2 test.

(a) (10 pts) The seven steps:

1. H_0 : The populations are homogeneous.
H_1 : The populations are not homogeneous.

2. $\alpha = 0.05$.

3. $\chi^2 = \sum_{\text{all cells}} \frac{(O - E)^2}{E}$.

4. $\chi^2 = 0.3333$.

5. p-value $= \chi^2$`cdf(0.3333,E99,2)` $= 0.8465$.

6. Accept H_0.

7. The populations are homogeneous.

(b) (2 pts) The expected values are shown in the following table.

Type of road	Interstate	Primary	Secondary
No. of crashes	765 (764.1)	3505 (3510.9)	1920 (1915.0)
No. of injuries	59 (59.9)	281 (275.1)	145 (150.0)

6. (12 pts)

(a) (8 pts) You need to enter the data into a 2×4 matrix and then run the χ^2 test. The seven steps:

1. H_0 : The populations are homogeneous.
H_1 : The populations are not homogeneous.

2. $\alpha = 0.05$.

3. $\chi^2 = \sum_{\text{all cells}} \frac{(O - E)^2}{E}$.

4. $\chi^2 = 5.378$.

5. p-value $= \chi^2$`cdf(5.378,E99,3)` $= 0.1461$.

6. Accept H_0.

7. The populations are homogeneous.

(b) (2 pts) The first row total is 152, the last column total is 2507, and the grand total is 8333, so the expected count in the first row, last column is

$$\frac{1152 \times 2507}{8333} = 45.7.$$

If you used the TI-83 χ^2`-Test`, you can check the matrix of expected counts and find the value 45.7 in the first row, last column.

7. (12 pts)

(a) (2 pts) One variable is whether the road is straight or curved. The other variable is whether the driver was speeding or not speeding.

(b) (8 pts) Put the data into a 2×2 matrix and use the χ^2 test. The problem was stated in terms of the independence of the variables. Therefore, the hypotheses should also be stated in those terms. The seven steps:

1. H_0 : The variables are independent.
H_1 : The variables are not independent.

2. $\alpha = 0.05$.

3. $\chi^2 = \sum_{\text{all cells}} \frac{(O-E)^2}{E}$.

4. $\chi^2 = 1814.5$.

5. p-value $= \chi^2$`cdf(1814.5,E99,1)` $= 0$.

6. Reject H_0.

7. The variables are not independent. (That is, speed is more likely to be a factor in a traffic death that occurs on a curved road than it is on a straight road.)

(c) (2 pts) The first row total is 13663, the first column total is 31498, and the grand total is 42398. So the expected count in row 1, column 1 is

$$\frac{13663 \times 31498}{42398} = 10150.4.$$

You can check the matrix of expected counts and see the value 10150.4 in row 1, column 1.