

1. (13 pts) For the sample data

72, 84, 93, 95, 97, 99, 99, 101, 106, 114, 126,

find the following statistics.

- (a) (5 pts) The mean
 - (b) (5 pts) The standard deviation
 - (c) (3 pts) The variance
2. (16 pts) For the data given in the previous problem, find the following statistics. (You may need the formula $r = 1 + \frac{p}{100}(n - 1)$.)
- (a) (4 pts) The median
 - (b) (4 pts) The first and third quartiles
 - (c) (4 pts) The 60th percentile
 - (d) (4 pts) The percentile rank of 88
3. (8 pts) Using the data in the first problem and your results in the second problem,
- (a) (5 pts) Draw a modified boxplot of the data.
 - (b) (3 pts) Based on your boxplot in part (a), describe the shape of the distribution of the data. Mention all important characteristics that can be inferred from the boxplot.
4. (20 pts) Suppose a set of scores on a statistics test has the distribution $N(77, 13)$. Let the random variable X represent a test score selected at random from the set. Find the following.
- (a) $P(X < 90)$
 - (b) $P(60 < X < 100)$
 - (c) The 60th percentile of X
 - (d) The Statistics Department's policy is that only students who score in the top 10% on this test are eligible for the department's annual award. What is the cutoff score that separates the top 10% from the rest of the class?

(over)

5. (18 pts) If a coin is tossed three times, the probability of getting 0 heads is $\frac{1}{8}$, the probability of 1 heads is $\frac{3}{8}$, the probability of 2 heads is $\frac{3}{8}$, and the probability of 3 heads is $\frac{1}{8}$.

Now consider the following game. A coin is tossed three times. The player wins \$5.00 if there are no heads, \$10.00 if there is one heads, \$15.00 if there are two heads, and he loses \$50.00 if there are three heads. Let the random variable X be the amount of money won when this game is played.

- (a) (5 pts) Write the pdf of X as a table.
 - (b) (5 pts) Find $E(X)$.
 - (c) (5 pts) Find the standard deviation of X .
 - (d) (3 pts) Given that the player won at least \$5.00, what is the probability that he won at least \$10.00?
6. (25 pts) At a restaurant the manager is concerned with the time it takes for a customer to receive his meal, once he orders. He will conduct a survey of one customer, measuring the waiting time. Let X be the waiting time. Based on the value he observes, he will choose between the two competing hypothetical models described as follows.

H_0 : X has a uniform distribution from 5 minutes to 10 minutes.

H_1 : X has a uniform distribution from 5 minutes to 20 minutes.

- (a) Sketch the pdfs of X under each of the two hypotheses, showing all relevant features.
- (b) If H_0 is true, what is the probability that a randomly selected customer waits at least 8 minutes for his meal?
- (c) In this part and the next, let the decision rule be to reject H_0 if $X > 9$. Shade the part of your diagram in part (a) that represents α , label it “ α ”, and then find the value of α .
- (d) Shade the part of your diagram in part (a) that represents β , label it “ β ”, and then find the value of β .
- (e) If the observed waiting time is 7.5 minutes, what is the p -value?