Math 121 **Test #3** **Fall 2007**

Grade distribution: $(32, 61, 76, 81, 93)$, $\overline{x} = 68.4$.

1. (12 pts) $(2, 7.25, 10.5, 11, 12)$, $\overline{x} = 9.1$.

   This is a one-sample-proportion hypothesis test. There was one sample of 700 people and the question has to do with proportions. The seven steps are

   Step 1. The hypotheses

   $H_0: \ p = 0.50$

   $H_1: \ p > 0.50$

   Step 2. $\alpha = 0.01$.

   Step 3. The test statistic is

   $$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}.$$

   Step 4. We are given that $\hat{p} = 0.58$ and that $n = 700$. So

   $$z = \frac{0.58 - 0.50}{\sqrt{\frac{0.50(1-0.50)}{700}}} = \frac{0.08}{0.01890} = 4.233.$$

   Step 5. The p-value is $P(Z > 4.233) =$ `normalcdf(4.233,E99)` $= 1.153 \times 10^{-5}$.

   Step 6. The p-value is less than α, so reject H_0.

   Step 7. Conclude that a majority of Virginians favor holding the line on taxes and reducing spending.

   In steps 4 and 5, you could use `1-PropZTest` with $\mathtt{p_0} = 0.50$, $\mathtt{x} = 0.58 \times 700 = 406$, $\mathtt{n} = 700$, and choose $\mathtt{>p_0}$.

2. (12 pts) $(2, 10.25, 11, 12, 12)$, $\overline{x} = 9.7$.

   This problem is a two-sample-proportion hypothesis test. There are two samples (one for Democrats and one for Republicans), each with a proportion. Let p_1 be the proportion of Democrats who consider themselves to be moderate and let p_2 be the proportion of Republicans who consider themselves to be moderate. The seven steps are

   Step 1. The hypotheses

   $H_0: \ p_1 = p_2$

   $H_1: \ p_1 > p_2$

   Step 2. $\alpha = 0.05$.

Step 3. The test statistic is

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}},$$

where $\hat{p}$ is the pooled estimate $\frac{x_1+x_2}{n_1+n_2}$.

Step 4. We are given that $\hat{p}_1 = 0.37$, $\hat{p}_2 = 0.28$, $n_1 = 1000$, and $n = 1000$. The pooled estimate is $\hat{p} = \frac{370+280}{1000+1000} = 0.325$. So

$$z = \frac{0.37 - 0.28}{\sqrt{0.325(1-0.325)\left(\frac{1}{1000} + \frac{1}{1000}\right)}} = \frac{0.09}{0.02095} = 4.297.$$

Step 5. The p-value is $P(Z > 4.297) =$ `normalcdf(4.297,E99)` $= 8.675 \times 10^{-6}$.

Step 6. The p-value is less than α, so reject H_0.

Step 7. Conclude that a greater proportion of Democrats than Republicans consider themselves to be moderate.

In steps 4 and 5, you could use `2-PropZTest` with $\texttt{x}_1 = 370$, $n_1 = 1000$, $\texttt{x}_2 = 280$, $\texttt{n}_2 = 1000$, and choose $\texttt{>p}_2$.

3. (8 pts) $(0, 7.25, 8, 8, 8)$, $\overline{x} = 6.8$.

   The formula is

$$(\hat{p}_1 - \hat{p}_2) \pm z\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}.$$

   That gives us

$$(0.37 - 0.28) \pm 1.645\sqrt{\frac{(0.37)(0.63)}{1000} + \frac{(0.28)(0.72)}{1000}} = 0.09 \pm 0.03430.$$

   The coefficient 1.645 is from `invNorm(0.05)`. You can use `2-PropZInt` on the TI-83 and get `(.05571,.12429)`.

4. (6 pts) $(2, 4, 4, 5.5, 6)$, $\overline{x} = 4.1$.

   (a) True. The rule is, reject H_0 if the p-value is less than α.

   (b) False. If the null hypothesis is true, then α is the probability that we will (mistakenly) reject H_0, regardless of the sample size.

   (c) False. A 95% confidence interval has a 95% chance of containing μ, regardless of the sample size. If the sample size is larger, then the interval will be narrower, but the narrower interval will still have a 95% chance of containing μ.

5. (8 pts) $(0, 0, 7, 8, 8)$, $\overline{x} = 4.7$.

   In both parts, use the `tcdf` function on the TI-83.

   (a) `tcdf(2.5,E99,3)` $= 0.04385$.

   (b) `tcdf(-1.0,1.0,25)` $= 0.6731$.

6. (16 pts) $(0, 10.25, 13.5, 14, 16)$, $\overline{x} = 11.4$.

   (a) (3 pts) You should use the normal distribution. That is because (1) the underlying population is normal and (2) you are using s instead of σ. It is also true that the sample size is small, but that is not a reason to use the t distribution.

   (b) (8 pts) Enter the data into the TI-83 and use `1-Var Stats` to find $\overline{x} = 71.92$ and $s = 10.73$. Then

   $$\overline{x} \pm t\frac{s}{\sqrt{n}} = 71.92 \pm 2.201\left(\frac{10.73}{12}\right) = 71.92 \pm 6.82.$$

   The value 2.201 comes from the t table, row 11, the 95% column.

   You can also use the function `TInterval` on the TI-83 and get the answer `(65.098,78.735)`.

   (c) (3 pts) The margin of error is 6.82. You can see it in the previous answer 71.92 ± 6.82. If you used the `TInterval` function in part (b), then you should take the difference $78.735 - 65.098 = 13.637$ and divide by 2 to get 6.8185.

   (d) (2 pts) The interval would be wider. That is because μ is more likely to be in a wider interval than in a narrower interval.

7. (12 pts) $(9, 5.5, 10, 11, 12)$, $\overline{x} = 8.1$.

   The seven steps are:

   Step 1. The hypotheses

   $H_0:\ \mu = 78.6$

   $H_1:\ \mu < 78.6$

   Step 2. $\alpha = 0.05$.

   Step 3. The test statistic is $t = \frac{\overline{x} - 78.6}{s/\sqrt{n}}$.

   Step 4. The value of t is

   $$t = \frac{71.92 - 78.6}{10.73/\sqrt{12}} = \frac{-6.68}{3.097} = -2.157.$$

   Step 5. The p-value is $P(t < -2.157) =$ `tcdf(-E99,-2.157,11)` $= 0.02701$.

   Step 6. The p-value is less than α, so reject H_0.

Step 7. We conclude that the average temperature in October over the past 30 years is less than $78.6°$.

You can use the function `T-Test` on the TI-83 and get $t = -2.157$ and p-value $=$ 0.02698.

8. (12 pts) $(2, 6.75, 9.5, 11.75, 12)$, $\overline{x} = 8.9$.

   Oops. A problem of this type was not supposed to be on the test. I got carried away. So, if your score on this problem improved your grade, I'll count it. If it hurt your grade, I'll drop it.

   The seven steps are:

   Step 1. Let μ_1 be the HSC average SAT score and μ_2 be the RMC average SAT score. The hypotheses are

   $H_0: \mu_1 = \mu_2$

   $H_1: \mu_1 > \mu_2$

   Step 2. $\alpha = 0.05$.

   Step 3. The test statistic is

$$t = \frac{\overline{x}_1 - \overline{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

   where s_p is the pooled estimate of σ.

   Step 4. The pooled estimate of σ is $s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{39\cdot 110^2+59\cdot 105^2}{98}} = \sqrt{11452.8} = 107.0$. The value of t is

$$t = \frac{1130 - 1086}{107.0\sqrt{\frac{1}{40} + \frac{1}{60}}} = \frac{44}{21.84} = 2.015.$$

   Step 5. Since the sample sizes are large, you could use either `normalcdf` or `tcdf` with 98 degrees of freedom. The results will be very similar.

$$p\text{-value} = P(t > 2.015) = 0.02332.$$

   Step 6. Reject H_0.

   Step 7. Conclude that the average SAT score of HSC students is higher than it is for RMC students.

   You could also use the functions `2-SampTTest` or `2-SampZTest` on the TI-83 and get $t = 2.014$ and p-value $= 0.02336$. It also reports the pooled estimate of σ as `Sxp` $= 107.02$.

9. (14 pts) $(0, 1.25, 3.5, 6, 12)$, $\overline{x} = 4.3$.

(a) According to the Central Limit Theorem (p. 556), the sampling distribution of $\overline{x}$ is normal with mean $\mu = 500$ and standard deviation $s/\sqrt{n} = 100/\sqrt{100} = 10$.

(b) The mean of the sampling distribution of $\overline{x}$ does not depend on n, so it does not change as n increases.

(c) The standard deviation of the sampling distribution of $\overline{x}$ is $s/\sqrt{n}$, so it decreases as n increases.

(d) Since $\overline{x}$ is $N(500, 10)$, then

$$P(475 < \overline{x} < 525) = \texttt{normalcdf(475,525,500,10)} = 0.9876.$$