Math 121 **Test #2 Solutions** **Spring 2007**

Grade distribution: $(31, 65, 70, 84, 91), \overline{x} = 71.4$.

1. (26 pts) $(18, 22.5, 24, 24.5, 26), \overline{x} = 23.5$.

   Enter the data into a list and store the list in `L1`. That is, enter

   `{32, 41, 36, 32, 37, 32, 35, 38, 25, 42, 35}→L1`

   Then press `STAT>CALC>1-Var Stats`. When `1-Var Stats` appears in the display, enter `L1` and press Enter. A list of statistics appears. You will find the statistics for (a) - (c) in that list.

   (a) (6 pts) The mean is 35

   (b) (6 pts) The standard deviation is 4.754. Because these numbers represent a sample, not the population, you should use `Sx`, not σ`x`.

   (c) (6 pts) The 5-number summary is min= 25, $Q_1 = 32$, median= 35, $Q_3 =$ 38, and max= 42.

   (d) (8 pts) It turns out that there are no outliers, so the modified boxplot will look the same as a basic boxplot. (I can't get my drawing program to draw it right now. Check back later.)

2. (6 pts) $(0, 3, 3, 3.5, 6), \overline{x} = 3.4$.

   Substitute into the formula $p = 85$ and $n = 11$. Then compute $r = 1+0.84(11-1) = 1 + 8.5 = 9.5$. So the 85^{th} percentile is halfway between the 9^{th} and 10^{th} numbers, when the numbers are arranged from smallest to largest. The 9^{th} number is 38 and the 10^{th} is 41, so the 85^{th} percentile is 39.5.

   You should not use the normal distribution (`invNorm`) for this problem because we were not told that the distribution is normal. It so happens that the distribution is close to normal, so `invNorm` gives a very close answer (39.695).

3. (9 pts) $(0, 8, 9, 9, 9), \overline{x} = 7.5$.

   (a) Use `normalcdf(-E99,-1.65)` and get 0.04947.

   (b) Use `normalcdf(-1.65,1.85)` and get 0.9184.

   (c) Use `normalcdf(1.85,E99)` and get 0.0.03216.

4. (4 pts) $(0, 4, 4, 4, 4), \overline{x} = 3.4$.

   Use `invNorm(.85)` and get 1.036.

5. (12 pts) $(0, 9, 12, 12, 12), \overline{x} = 9.7$.

   Let the distribution of the random variable X be $N(30, 5)$. Find the following.

   (a) Enter `normalcdf(-E99,25,30,5)` and get 0.1587.

(b) Enter `normalcdf(22,32,30,5)` and get 0.6006.

(c) Enter `normalcdf(44,E99,30,5)` and get 0.002555.

(d) Enter `invNorm(.45,30,5)` and get 29.37.

6. (9 pts) $(0, 1.5, 3, 7, 9), \overline{x} = 4.1$.

   (a) (3 pts) We know that $A = \frac{1}{2}bh$ and that $A = 1$ and $b = 6$. Therefore, $1 = 3h$. It follows that $h = \frac{1}{3}$.

   (b) (6 pts) The point 1 is $\frac{1}{3}$ the way from 0 to 3 (the peak). Therefore, the height at 1 is $\frac{1}{3}$ of the peak height of $\frac{1}{3}$, i.e., the height is $\frac{1}{9}$. So the area of the triangle to the left of 1 is $\left(\frac{1}{2}\right)(1)\left(\frac{1}{9}\right) = \frac{1}{18}$.

7. (10 pts) $(3, 6.5, 8, 10, 10), \overline{x} = 7.8$.

   (a) (5 pts) The direction of extreme is to the *right*, so the value of α is the area to the *right* of 4 in the H_0 picture. That is $\frac{1}{5} = 0.20$.

   (b) (5 pts) The value of β is the area to the *left* of 4 in the H_1 picture. That is $\frac{2}{5} = 0.40$.

8. (12 pts) $(0, 2.5, 5, 7, 10), \overline{x} = 4.6$.

   (a) (3 pts) In a sample of size 2, we could have 0, 1, or 2 people who commute by public transportation. So the possible sample proportions are $\frac{0}{2} = 0$, $\frac{1}{2}$, and $\frac{2}{2} = 1$.

   (b) (6 pts) The tree diagram shows that the probability is $\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$ of getting 2 people who commute by public transportion, $2 \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{4}{9}$ of getting just 1 such person, and $\frac{2}{9} \cdot \frac{2}{9} = \frac{4}{9}$ of getting no such person. So the sampling distribution is

$\hat{p}$	Prob.
0	4/9
1/2	4/9
1	1/9

   (c) (3 pts) According to the Central Limit Theorem for Proportions, the mean of $\hat{p}$ is $\mu_{\hat{p}} = p = \frac{1}{3}$ and the standard deviation is $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{\frac{1}{3}\cdot\frac{2}{3}}{2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$.

9. (12 pts) $(0, 4, 8, 11, 12), \overline{x} = 7.3$.

   (a) (4 pts) According to the Central Limit Theorem for Means, the mean of $\overline{x}$ is $\mu_{\overline{x}} = \mu = 2.5$, the standard deviation is $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.443}{\sqrt{100}} = 0.1443$, and the shape is normal, because $n > 30$.

(b) (8 pts) Use the information in part (a) to find the probability. The probability is the area between 2.25 and 2.75 under the normal curve with mean 2.5 and standard deviation 0.1443. Enter `normalcdf(2.25,2.75,2.5,0.1443)` and get 0.9168.