# Math 121 — Test #3 Solutions — Spring 2007

Grade distribution: $(22, 63, 82, 93, 97), \overline{x} = 73.4$.

1. (15 pts) $(2, 6, 9.5, 12.75, 15), \overline{x} = 9.2$.

   This is a test comparing a single proportion $p$ to 0.50. Therefore, you should use `1-PropZTest` on the TI-83. Do not be confused by the *two* numbers 542 and 458. There is only *one* sample and, therefore, only one proportion.

   There is no indication that the teacher expects the coin to be biased in favor of heads. It so happens that he got more heads than tails, but that does not mean that he was testing only for bias in favor of heads. Therefore, it should be a two-tailed test.

   1. $H_0$: $p = 0.50$
      $H_1$: $p \neq 0.50$
      where $p$ is the proportion of heads.
   2. $\alpha = 0.01$.
   3. $Z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$.
   4. We have $\hat{p} = \frac{542}{1000} = 0.542$, $p_0 = 0.50$, and $n = 1000$. We calculate
      $$Z = \frac{0.542 - 0.50}{\sqrt{\frac{0.50(1-0.50)}{1000}}} = \frac{0.042}{0.01581} = 2.656.$$
   5. $p$-value $= 2 \times$ `normalcdf(2.656,E99)` $= 0.007907$.
   6. Reject $H_0$ because the $p$-value is less than $\alpha$.
   7. We conclude that the coin is biased.

2. (15 pts) $(0, 13.5, 15, 15, 15), \overline{x} = 11.9$.

   This is a confidence interval estimating a single proportion $p$. You should use `1-PropZInt` on the TI-83. You must calculate the value of `x`, i.e., the *number* of people who said that gun control will not be an issue. Compute `x` $= 0.58 \times 216000 = 125280$. The sample size is $n = 216000$ and the confidence level is 0.95. The TI-83 reports the confidence interval to be `(.57792,.58208)`.

   If you do it "by hand," you evaluate the formula $\hat{p} \pm z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$. Compute the confidence interval to be
   $$0.58 \pm 1.96\sqrt{\frac{(0.58)(0.42)}{216000}} = 0.58 \pm 0.00208.$$

3. (15 pts) $(3, 10.5, 13.5, 15, 15), \overline{x} = 11.9$.

   In this problem you are to test a hypothesis concerning a single mean $\mu$. The sample size is small, we are sampling from a normal population, and $\sigma$ is unknown. Therefore, we must use the $t$ distribution and the function `T-Test` on the TI-83.

   Enter the data into a list: `{28,29,34,24,22,29,35,28,28,33}`→$L_1$. The hypothetical mean $\mu_0$ is 32. Use `T-Test` and select `Data` to get the value of $t$ and the $p$-value. The seven steps are

   1. $H_0$: $\mu = 32$
      $H_1$: $\mu < 32$
   2. $\alpha = 0.01$.
   3. $t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$.
   4. You may use `T-Test` or use `1-Var Stats` $L_1$ and get $\overline{x} = 29$, $s = 4.137$, and $n = 10$. Evaluate the formula and get
      $$t = \frac{29 - 32}{4.137/\sqrt{10}} = \frac{-3}{1.308} = -2.293.$$
   5. $p$-value = `tcdf(-E99,-2.293,9)` = 0.02376. (There are 9 degrees of freedom.)
   6. Accept $H_0$.
   7. Conclude that the average fat content is not less than 32 g.

4. (20 pts) $(0, 10, 18, 20, 20), \overline{x} = 13.4$.

   (a) (10 pts) $(0, 7.25, 10, 10, 10), \overline{x} = 7.8$.

   Use the formula $\overline{x} \pm t\left(\dfrac{s}{\sqrt{n}}\right)$ with the values given in the previous problem. Compute
   $$29 \pm 1.833\left(\frac{4.137}{\sqrt{10}}\right) = 29 \pm 2.398.$$
   You can use the TI-83 function `TInterval`. It will give the result `(26.602,31.398)`, which is the same interval as above.

   (b) (5 pts) $(0, 0, 4, 5, 5), \overline{x} = 2.8$.

   If you used the formula, then you can see that the margin of error is 2.398. If you used `T-Interval`, then the margin is half the difference between the two endpoints: $(0.5)(31.398 - 26.602) = 2.398$.

   (c) (5 pts) $(0, 0, 5, 5, 5), \overline{x} = 2.9$.

   A 95% confidence interval would have a larger margin of error. That is why it is more likely to contain $\mu$. Or consider how the calculation would differ. All the values would be the same except $t$. The $t$ value would be 2.262 instead of 1.833, giving a wider interval. Or, use `T-Interval` to find the 95% confidence interval and you will see that it is wider.

5. (20 pts) $(4, 15, 18, 19, 20), \overline{x} = 15.7$.

   (a) (15 pts) $(4, 12, 14, 14, 15), \overline{x} = 11.9$.

   This is a test comparing the means $\mu_1$ and $\mu_2$ of two different populations. Let $\mu_1$ be the mean hourly wages in metropolitan areas and $\mu_2$ be the mean hourly wages in non-metropolitan areas.

   1. $H_0$: $\mu_1 = \mu_2$
      $H_1$: $\mu_1 > \mu_2$
   2. $\alpha = 0.05$.
   3. The sample sizes are large, so we could use

      $$Z = \frac{\overline{x}_1 - \overline{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

      but $\sigma$ is unknown, so we could also use

      $$T = \frac{\overline{x}_1 - \overline{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

      with 198 degrees of freedom.
   4. First compute the pooled estimate for $\sigma$.

      $$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{99(3.22)^2 + 99(8.37)^2}{198}} = 6.341.$$

      Then

      $$t = \frac{21.44 - 16.74}{6.341\sqrt{\frac{1}{100} + \frac{1}{100}}} = \frac{4.70}{0.8968} = 5.241.$$
   5. $p$-value = `tcdf(5.241,E99,198)` $= 2.038 \times 10^{-7} = 0.0000002038$.
   6. Reject $H_0$.
   7. The metropolitan hourly wages are greater than the non-metropolitan hourly wages.

   This problem could be worked on the TI-83 using either `2-SampZTest` or `2-SampTTest`.

   (b) (5 pts) $(0, 2.25, 5, 5, 5), \overline{x} = 3.7$.

   The calculations in part (a) show that $s_p = 6.341$. If you use `2-SampTTest`, the calculator will report the same thing.

6. (15 pts) $(0, 8.75, 12.5, 15, 15), \overline{x} = 11.2$.

   This is a test comparing proportions $p_1$ and $p_2$ from two different populations. Let $p_1$ be the proportion of the male population that smokes and $p_2$ be the proportion of the female population that smokes.

1. $H_0$: $p_1 = p_2$
   $H_1$: $p_1 > p_2$
2. $\alpha = 0.05$.
3. $Z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$, where $\hat{p}$ is the pooled estimate for $p$.
4. First find that there were $0.28 \times 1000 = 280$ men in sample 1 who smoked and $0.24 \times 600 = 144$ women in sample 2 who smoked. Then $\hat{p} = \frac{280+144}{1000+600} = \frac{424}{1600} = 0.265$. Then compute

$$z = \frac{0.28 - 0.24}{\sqrt{(0.265)(0.735)\left(\frac{1}{1000} + \frac{1}{600}\right)}} = \frac{0.04}{0.02279} = 1.755.$$

5. $p$-value = `normalcdf(1.755,E99)` $= 0.03963$.
6. Reject $H_0$.
7. The proportion of males who smoke is greater than the proportion of females who smoke.