Math 121 **Test #2** **Spring 2009**

1. (10 pts)

   (a) (6 pts) Let the population be the HSC faculty of the rank Professor and let the variable of interest be the number of years that they have been teaching at HSC. A five-number summary of the values of this variable is

   $$7 \quad 19 \quad 25 \quad 29.5 \quad 45.$$

   Draw a basic boxplot for this variable.

   (b) (4 pts) Now let the population be all regular HSC faculty at HSC and let the variable continue to be the number of years that they have been teaching here. Then the five-number summary is

   $$1 \quad 7 \quad 14 \quad 24 \quad 45.$$

   Use this five-number summary to describe the shape of the distribution.

2. (12 pts) A computer program was tested to determine its run time. The program uses random data, so the run time is unpredictable. The program was tested 10 times and the run times, in microseconds, were

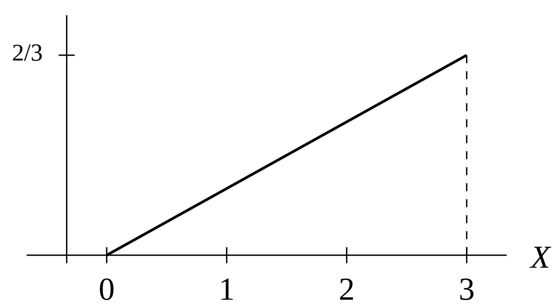
   $$8.76 \quad 8.79 \quad 8.80 \quad 8.82 \quad 8.85 \quad 8.85 \quad 8.86 \quad 8.86 \quad 8.87 \quad 8.94.$$

   (a) (4 pts) Find the mean run time.

   (b) (4 pts) Find the median run time.

   (c) (4 pts) Find the standard deviation of the run times.

3. (10 pts) A random variable X has the triangular distribution shown in the figure below. Determine the probability that a randomly selected value of that variable is between 0 and 1.



4. (18 pts) Suppose that the variable of interest is the waiting time at the check-out line at a grocery store. The null hypothesis says that it has a uniform distribution from 0 to 6 minutes, while the alternative hypothesis says that it is uniform from 0 to 15 minutes. To test these hypotheses, the waiting time of a single customer is observed. If it is 5 minutes or longer, the null hypothesis will be rejected.

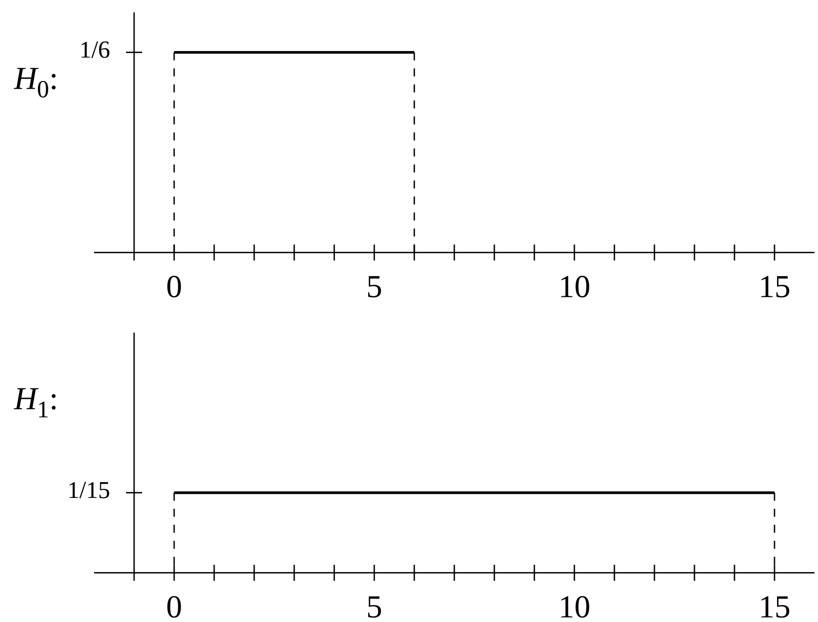
(a) (2 pts) What is the direction of extreme?

(b) (6 pts) Find the values of α and β for this test. In the figures below, shade and label the regions that represent α and β.

(c) (4 pts) If the null hypothesis is true, what is the probability that a customer will wait from 2 to 5 minutes?

(d) (4 pts) If the alternative hypothesis is true, what is the probability that a customer will wait from 2 to 5 minutes?

(e) (2 pts) Explain why the height of the alternative graph was set at $\frac{1}{15}$ while the height of the null graph was set at $\frac{1}{6}$?



5. (12 pts) Let Z be the standard normal random variable and let X be a normal random variable representing IQ scores with distribution $N(100, 15)$. Find the following.

(a) $P(-1.5 < Z < 1.5)$.

(b) $P(Z > 2.5)$.

(c) $P(82 < X < 120)$.

(d) The 15^{th} percentile of Z.

(e) The 95^{th} percentile of X.

(f) The two values of X that determine the middle 20%.

6. (10 pts) Assume that cholesterol levels in adult Americans is normally distributed with a mean of 190 and a standard deviation of 20.

   (a) (5 pts) What is the probability that a randomly selected adult American has a cholesterol level less than 175?

   (b) (5 pts) Approximately what percentage of all adult Americans have cholesterol within one standard deviation of the mean?

7. (10 pts) At a certain university, 30% of the students are freshmen. Let $\hat{p}$ represent the sample proportion of freshmen in samples of size $n = 84$.

   (a) (4 pts) Use the Central Limit Theorem to describe the sampling distribution of $\hat{p}$.

   (b) (6 pts) What is the probability that the sample proportion of freshmen in a random sample is at least 20%?

8. (10 pts) Let X be the variable in problem 3. It is a fact that the mean of X is $\mu = 2$ and the standard deviation of X is $\sigma = \frac{1}{\sqrt{2}} = 0.7071$. Let $\overline{x}$ represent the sample mean of samples of size $n = 32$.

   (a) (4 pts) Use the Central Limit Theorem to describe the sampling distribution of $\overline{x}$.

   (b) (6 pts) What is the probability that the sample mean $\overline{x}$ of a random sample is between 1.8 and 2.2?

9. (8 pts) A box contains a large number of vouchers. One-third of them are worth \$5, one-third are worth \$10, and one-third are worth \$15. Two vouchers are selected at random (with replacement). Let $\overline{x}$ represent the average value of the two selected vouchers. Find the sampling distribution of $\overline{x}$.