Math 121 **Test #3 Solutions** **Spring 2012**

1. (10 pts) For a population with mean μ and standard deviation σ, the sampling distribution of $\overline{x}$ of all possible samples of size n has mean μ and standard deviation $\dfrac{\sigma}{\sqrt{n}}$. Furthermore, if the sampled population is normal, then $\overline{x}$ is normal for any sample size n. If the sampled population is not normal, then $\overline{x}$ is approximately normal provided the sample size is at least 30.

2. (12 pts) This problem involves a proportion of a single sample. Show all seven steps.

   (1) Let p be the proportion of 20- to 34-year olds who had drivers licenses in 2010. The hypotheses are

   $H_0: \quad p = 0.80$
   $H_1: \quad p > 0.80$

   (2) $\alpha = 0.05$.

   (3) Let $z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$.

   (4) The sample proportion is 0.843 or $\frac{337}{400} = 0.8425$. Use `1-PropZTest` or compute

   $$\begin{aligned} z &= \frac{.8425 - 0.80}{\sqrt{\frac{(0.80)(0.20)}{400}}} \\ &= \frac{0.425}{0.02} \\ &= 2.125. \end{aligned}$$

   (5) p-value = `normalcdf(2.125,E99)` = 0.0168.

   (6) Reject H_0 because the p-value is less than α.

   (7) The proportion of 20- to 34-year-olds with drivers licenses in 2010 was more than 80%.

3. (10 pts) Use `1-PropZInt` and get $(0.8068, 0.8782)$ or compute

   $$\begin{aligned} \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} &= 0.8425 \pm 1.960\sqrt{\frac{(0.8425)(0.1575)}{400}} \\ &= 0.8425 \pm 0.0357. \end{aligned}$$

4. (17 pts)

   (a) (14 pts) This is a hypothesis test of proportions involving two samples. Show the seven steps.

(1) Let p_1 be the proportion of 17-year-olds with drivers licenses in 1983 and let p_2 be the proportion of 17-year-olds with drivers licenses in 2008. The hypotheses are

$H_0: \quad p_1 = p_2$
$H_1: \quad p_1 > p_2$

(2) $\alpha = 0.05$.

(3) Let $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$.

(4) Use `2-PropZTest` or compute the following. First, the pooled estimate for p.

$$\begin{aligned}\hat{p} &= \frac{x_1 + x_2}{n_1 + n_2} \\ &= \frac{69 + 25}{100 + 50} \\ &= 0.6267.\end{aligned}$$

Then compute z.

$$\begin{aligned}z &= \frac{0.69 - 0.50}{\sqrt{(0.6267)(0.3733)\left(\frac{1}{100} + \frac{1}{50}\right)}} \\ &= \frac{0.19}{0.08377} \\ &= 2.268.\end{aligned}$$

(5) p-value $=$ `normalcdf(2.268,E99)` $= 0.01167$.

(6) Reject H_0 because the p-value is less than α.

(7) The proportion of 17-year-olds with drivers licenses in 1983 was greater than the proportion of 17-year-olds with drivers licenses in 2008.

(b) In part (a) the pooled estimate for p was calculated in Step 4. It also appears in the TI-83 display when you use `2-PropZTest`. The value is 0.6267.

5. (12 pts) This problem is a hypothesis test involving a mean of one sample. First, enter the data (2nd row) into the TI-83. Show the seven steps.

(1) Let μ be the mean number of trees of diameter 6" or greater of all such plots. The hypotheses are

$H_0: \quad \mu = 20$
$H_1: \quad \mu > 20$

(2) $\alpha = 0.10$.

(3) Because the sample is small and the population is (apparently) normal, we must use the t test.

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}.$$

(4) Use `T-Test` or compute the following. First, use `1-Var Stats` to get $\overline{x} = 22.75$ and $s = 9.603$. Then compute

$$\begin{aligned} t &= \frac{22.75 - 20}{9.603/\sqrt{8}} \\ &= \frac{2.75}{3.395} \\ &= 0.8100. \end{aligned}$$

(5) There are 7 degrees of freedom, so p-value = `tcdf(0.8100,E99,7)` = 0.2223.

(6) Accept H_0 because the p-value is greater than α.

(7) The average number of trees of diameter 6" or greater per plot is 20.

6. (10 pts) Use `TInterval` and get $(16.318, 29.182)$ or compute the following. First, use the t table to find the value of t for a 90% confidence with 7 degrees of freedom. The value is $t = 1.895$.

$$\begin{aligned} \overline{x} \pm t\left(\frac{s}{\sqrt{n}}\right) &= 22.75 \pm 1.895\left(\frac{9.603}{\sqrt{8}}\right) \\ &= 22.75 \pm 6.434. \end{aligned}$$

7. (17 pts)

(a) This is a hypothesis test involving means of two samples. This situation cannot be a paired sample because the sample sizes are different and because there is no sensible way to pair a rat on Diet 1 with a rat on Diet 2. Show the seven steps.

(1) Let μ_1 be the mean weight of rats on Diet 1 and let μ_2 be the mean weight of rats on Diet 2. The hypotheses are

$$\begin{aligned} H_0 &: \quad \mu_1 = \mu_2 \\ H_1 &: \quad \mu_1 \neq \mu_2 \end{aligned}$$

(2) $\alpha = 0.05$.

(3) The sample sizes are large, so it is ok to use z, but it is better to use t. In any case, we do not know σ_1 or σ_2, so we must use s_1 and s_2 in their place. Using t, the test statistic is

$$t = \frac{\overline{x}_1 - \overline{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}.$$

(4) Use `2-SampTTest` or compute the following. First, compute s_p and get

$$s_p = \sqrt{\frac{(59)(1.25)^2 + (79)(0.95)^2}{138}} = 1.0884.$$

Then compute

$$\begin{aligned} t &= \frac{8.4 - 9.0}{1.0884\sqrt{\frac{1}{60} + \frac{1}{80}}} \\ &= -\frac{0.6}{0.1859} \\ &= -3.228. \end{aligned}$$

(5) There are 138 degrees of freedom and the test is two-tailed, so

p-value $= 2 \times$ `tcdf(-E99,-3.228,138)` $= 0.001558.$

(6) Reject H_0 because the p-value is less than α.

(7) The weight of the rats on Diet 1 is different from the weight of the rats on Diet 2.

8. (12 pts) (Optional)

(a) According to the Central Limit Theorem, the values of $\overline{x}$ from samples of size $n = 25$ will have mean 69.5 and standard deviation $\dfrac{2.9}{\sqrt{25}} = 0.58$. Furthermore, because we are sampling from a population that is already normal, $\overline{x}$ will be normal (even though the sample size is small).

(b) Using the description in part (a), we calculate the probability as

`normalcdf(70,E99,69.5,0.58)` $= 0.1943.$