Infinite Limits

Lecture 10
Section 1.5

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Announcement

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- Test #1 is one week from today, Friday, February 3.

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- Be there.

Objectives

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- Limits of polynomials.
- Limits of rational functions.
- Limits involving square roots.
- Find limits “at infinity.”

Polynomials

Limits of Polynomials

If $f(x)$ is a polynomial and c is any real number, then

$$\lim_{x \to c} f(x) = f(c).$$

Limits of Rational Functions

Let $p(x)$ and $q(x)$ be polynomials and c be any real number. To find

$$\lim_{x \to c} \frac{p(x)}{q(x)},$$

Limits of Rational Functions

Let $p(x)$ and $q(x)$ be polynomials and c be any real number. To find $\lim_{x \to c} \frac{p(x)}{q(x)}$,

(1) If $q(c) \neq 0$, then

$$\lim_{x \to c} \frac{p(x)}{q(x)} = \frac{p(c)}{q(c)}.$$

Limits of Rational Functions

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(2) If $q(c) = 0$ and $p(c) \neq 0$, then $\lim_{x \to c} \dfrac{p(x)}{q(x)}$ does not exist.

Rational Functions

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(2) If $q(c) = 0$ and $p(c) \neq 0$, then $\lim_{x \to c} \frac{p(x)}{q(x)}$ does not exist.

(3) If $q(c) = 0$ and $p(c) = 0$, then factor $(x - c)$ out from both $p(x)$ and $q(x)$, cancel the factors, and re-evaluate the limit.

Limit at Infinity

Example 1.5.7

Find $\lim_{x \to \infty} \dfrac{x^2}{1 + x + 2x^2}$.

Limit at Infinity

Two Useful Facts

If $k > 0$, then

$$\lim_{x \to \infty} \frac{1}{x^k} = 0,$$
$$\lim_{x \to -\infty} \frac{1}{x^k} = 0,$$

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- Then take the limit, using the two rules above. The possibilities are

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  (3) $\lim_{x\to\infty} g(x) = 0$ and $\lim_{x\to\infty} f(x) = 0$. The limit may or may not exist.