Infinite Limits

Lecture 10
Section 1.5

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Fri, Feb 3, 2017

Reminder

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- Test #1 is this Friday, February 10.

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- Be there.

Objectives

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- Compute limits at infinity.
- Compute one-sided limits.
- Determine whether a function is continuous.
- Locate points of discontinuity.

Limit at Infinity

Example 1.5.7

Find $\lim_{x \to \infty} \frac{x^2}{1 + x + 2x^2}$.

Limit at Infinity

Two Useful Facts

If $k > 0$, then

$$\lim_{x \to \infty} \frac{1}{x^k} = 0,$$
$$\lim_{x \to -\infty} \frac{1}{x^k} = 0,$$

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- Then take the limit, using the two rules above. The possibilities are

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(2) $\lim_{x\to\infty} g(x) = 0$ and $\lim_{x\to\infty} f(x) \neq 0$. The limit does not exist.

(3) $\lim_{x\to\infty} g(x) = 0$ and $\lim_{x\to\infty} f(x) = 0$. The limit may or may not exist.

One-Sided Limits

Definition (Limit from the Left)

Let $f(x)$ be a function and c be a real number. The **limit from the left** of $f(x)$ as x approaches c is the value, if it exists, that $f(x)$ gets closer and closer to as x gets closer and closer to c, but always **less than** c. This limit is denoted

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Examples

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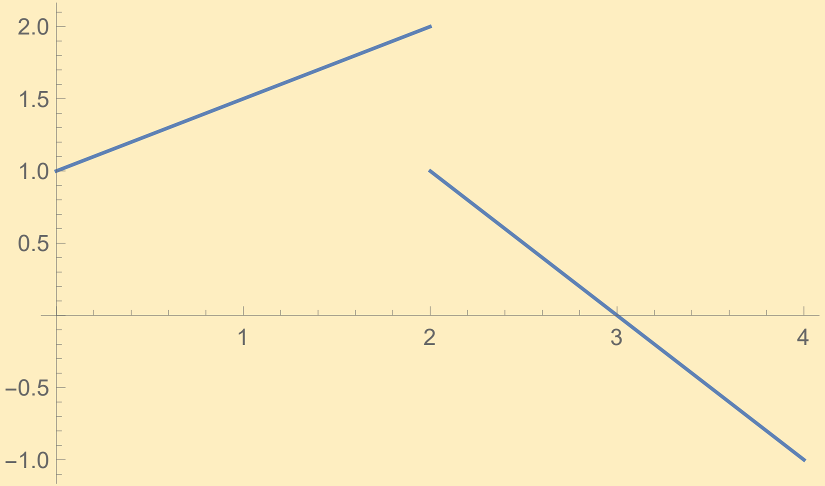
- $f(x) = \begin{cases} \frac{1}{2}x + 1 & \text{if } x \le 2 \\ 3 - x & \text{if } x > 2. \end{cases}$, at $c = 2$.
- $f(x) = \begin{cases} x^2 + 4x - 2 & \text{if } x \le 1 \\ -5x^2 + 8x & \text{if } x > 1. \end{cases}$, at $c = 1$.
- $f(x) = \dfrac{2x - 1}{x - 2}$, at $c = 2$.

Where is $f(x) = \begin{cases} \frac{1}{2}x + 1 & \text{if } x \le 2 \\ 3 - x & \text{if } x > 2. \end{cases}$ continuous?



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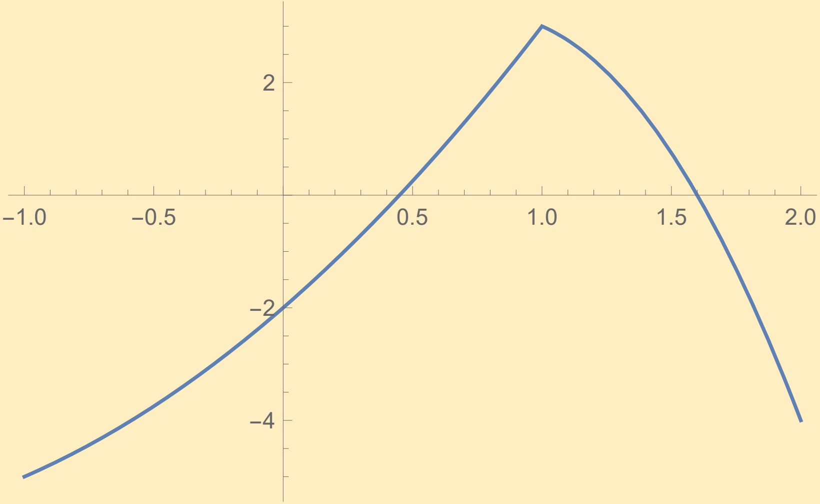
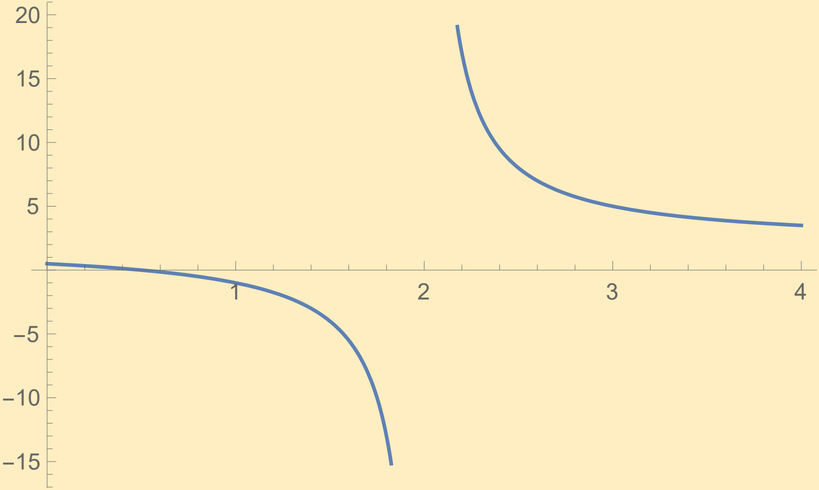


Where is $f(x) = \dfrac{2x - 1}{x - 2}$ continuous?



Two-Sided Limits

Definition (Two-Sided Limit)

Let $f(x)$ be a function and let c be a real number. Then $\lim\limits_{x \to c} f(x)$ exists if

(1) $\lim\limits_{x \to c^-} f(x)$ exists,

(2) $\lim\limits_{x \to c^+} f(x)$ exists, and

(3) $\lim\limits_{x \to c^-} f(x) = \lim\limits_{x \to c^+} f(x)$.

Continuity

Definition (Continuity)

Let $f(x)$ be a function and let c be a real number. Then $f(x)$ is **continuous** at c if

(1) $\lim_{x \to c} f(x)$ exists,

(2) $f(c)$ exists, and

(3) $\lim_{x \to c} f(x) = f(c)$.