

Instantaneous Rate of Change

Lecture 13
Section 2.1

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Hampden-Sydney College

Wed, Feb 8, 2017

Reminder

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- Test #1 is this Friday, February 10.

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- Be there.

Objectives

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- The concept of instantaneous rate of change.
- The definition of the derivative.
- Using the definition to find a derivative.

Instantaneous Rate of Change

Definition (Instantaneous Rate of Change)

The **instantaneous rate of change** of $f(x)$ with respect to x at a point c is

$$\lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}.$$

Instantaneous Rate of Change

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Let the cost function be

$$C(x) = 20x + 8000$$

where x is the number of units produced. Find the instantaneous rate of change of cost relative to production when production is $x = 100$.

Instantaneous Rate of Change

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Let the cost function be

$$C(x) = x^2 + 20x + 8000$$

where x is the number of units produced. Find the instantaneous rate of change of cost relative to production when production

- (a) is $x = 100$.
- (b) is $x = 200$.
- (c) is $x = 300$.

The Derivative

Definition (The Derivative)

The **derivative** of $f(x)$ with respect to x is the function

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

The Derivative

The Derivative

Let the cost function be

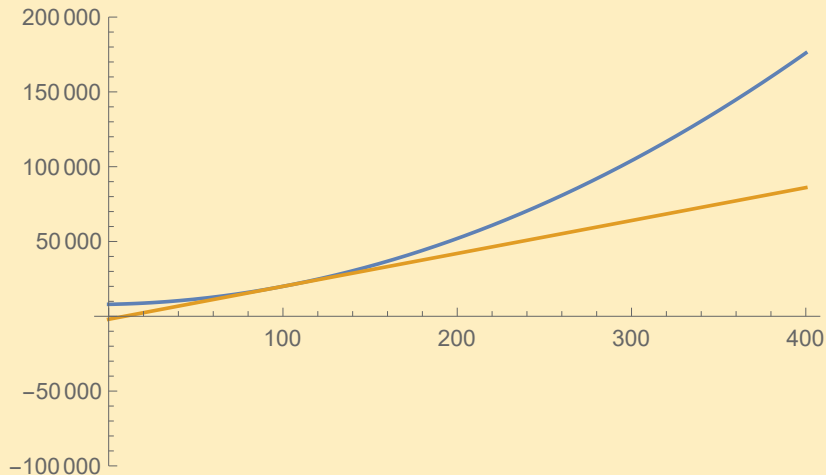
$$C(x) = x^2 + 20x + 8000$$

where x is the number of units produced.

- (a) Find the derivative $C'(x)$ of cost with respect to production.
- (b) Evaluate $C'(x)$ at $x = 100$, $x = 200$, and $x = 300$.

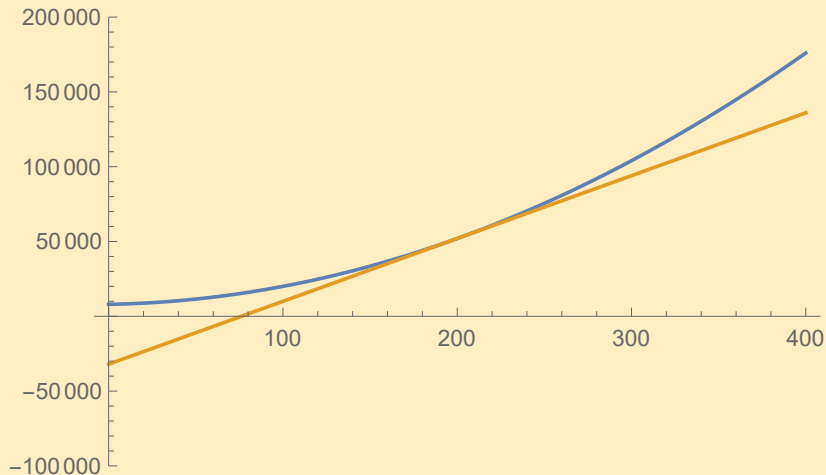
Tangent line to $C(x)$ at $x = 100$

Tangent line to $C(x)$ at $x = 100$



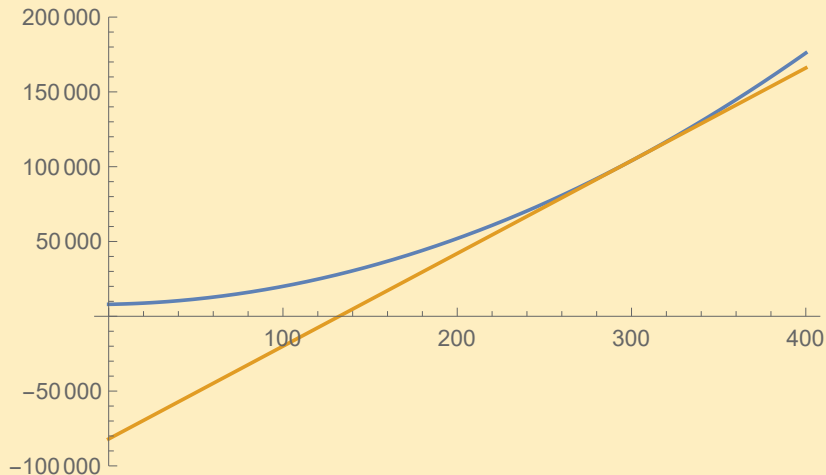
Tangent line to $C(x)$ at $x = 200$

Tangent line to $C(x)$ at $x = 200$



Tangent line to $C(x)$ at $x = 300$

Tangent line to $C(x)$ at $x = 300$



Example 2.1.5

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Gordon owns a small manufacturing firm. He determines that when x thousand units of one of his products are produced and sold, the profit generated will be

$$P(x) = -400x^2 + 6,800x - 12,000$$

dollars.

(a) Is production profitable when 9,000 units are produced?

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- (a) Is production profitable when 9,000 units are produced?
- (b) At what rate should Gordon expect profit to be changing with respect to the level of production x when 9,000 units are produced?

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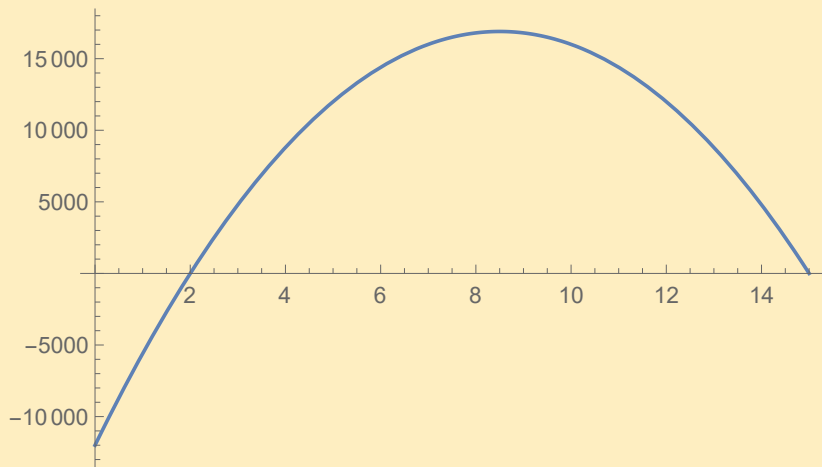
$$P(x) = -400x^2 + 6,800x - 12,000$$

dollars.

- (a) Is production profitable when 9,000 units are produced?
- (b) At what rate should Gordon expect profit to be changing with respect to the level of production x when 9,000 units are produced?
- (c) Is the profit increasing or decreasing at this level of production?

Example 2.1.5

Graph of $C(x) = -400x^2 + 6800x - 12000$



Example 2.1.5

Tangent Line to $C(x) = -400x^2 + 6800x - 12000$

