Maximums and Minimums

Lecture 25
Section 3.1

Robb T. Koether

Hampden-Sydney College

Mon, Mar 6, 2017

Objectives

Objectives

- Understand the difference between a relative extreme and an absolute extreme.
- Use the derivative to find the maximum and minimum values of a function.

Absolute Maximum

Definition (Absolute Maximum)

Let $f(x)$ be a function defined on an interval. The **absolute maximum** of $f(x)$ on that interval is the largest value of $f(x)$ for all x in that interval.

Absolute Maximum

Definition (Absolute Maximum)

Let $f(x)$ be a function defined on an interval. The **absolute maximum** of $f(x)$ on that interval is the largest value of $f(x)$ for all x in that interval.

- In other words, if M is the absolute maximum, then $f(x) \leq M$ for all x in the interval.

Absolute Maximum

Definition (Absolute Maximum)

Let $f(x)$ be a function defined on an interval. The **absolute maximum** of $f(x)$ on that interval is the largest value of $f(x)$ for all x in that interval.

- In other words, if M is the absolute maximum, then $f(x) \leq M$ for all x in the interval.
- A similar definition applies to the **absolute minimum.**

Relative Maximum

Definition (Relative Maximum)

Let $f(x)$ be a function defined on an interval. A **relative maximum** of $f(x)$ on that interval is a value of $f(x)$ that is the largest value among all "nearby" values.

Relative Maximum

Definition (Relative Maximum)

Let $f(x)$ be a function defined on an interval. A **relative maximum** of $f(x)$ on that interval is a value of $f(x)$ that is the largest value among all “nearby” values.

- In other words, if M is a relative maximum if there is some interval (however small) such that $f(x) \leq M$ for all x in that smaller interval.

Relative Maximum

Definition (Relative Maximum)

Let $f(x)$ be a function defined on an interval. A **relative maximum** of $f(x)$ on that interval is a value of $f(x)$ that is the largest value among all “nearby” values.

- In other words, if M is a relative maximum if there is some interval (however small) such that $f(x) \leq M$ for all x in that smaller interval.
- A similar definition applies to the **relative minimum**.

Critical Point

Definition (Critical Point)

A **critical point** of a function $f(x)$ is a point where $f'(x) = 0$ or where $f'(x)$ does not exist.

Location of Extreme Values

Theorem (Location of Extreme Values)

The extreme values of a function must occur at critical points of the function, but not every critical point need be the location of an extreme value.

Locating Extreme Values

Procedure Using the First Derivative for Finding Extreme Values

- Find $f'(x)$.

Locating Extreme Values

Procedure Using the First Derivative for Finding Extreme Values

- Find $f'(x)$.
- Solve the equation $f'(x) = 0$ to find those **critical points**.

Locating Extreme Values

Procedure Using the First Derivative for Finding Extreme Values

- Find $f'(x)$.
- Solve the equation $f'(x) = 0$ to find those **critical points**.
- Include as **additional critical points** all points where $f'(x)$ does not exist (for whatever reason).

Locating Extreme Values

Procedure Using the First Derivative for Finding Extreme Values

- Find $f'(x)$.
- Solve the equation $f'(x) = 0$ to find those **critical points**.
- Include as **additional critical points** all points where $f'(x)$ does not exist (for whatever reason).
- Choose **test points** between adjacent critical points.

Locating Extreme Values

Procedure Using the First Derivative for Finding Extreme Values

- Find $f'(x)$.
- Solve the equation $f'(x) = 0$ to find those **critical points**.
- Include as **additional critical points** all points where $f'(x)$ does not exist (for whatever reason).
- Choose **test points** between adjacent critical points.
- Evaluate the sign ($+$ or $-$) of $f'(x)$ at each test point.

Locating Extreme Values

Procedure Using the First Derivative for Finding Extreme Values

- Find $f'(x)$.
- Solve the equation $f'(x) = 0$ to find those **critical points.**
- Include as **additional critical points** all points where $f'(x)$ does not exist (for whatever reason).
- Choose **test points** between adjacent critical points.
- Evaluate the sign ($+$ or $-$) of $f'(x)$ at each test point.
- A **relative maximum** occurs at a critical point c if
  - $f(c)$ exists and
  - $f'(x) > 0$ at the test point to the left of c (increasing) and
  - $f'(x) < 0$ at the test point to the right of c (decreasing).

Locating Extreme Values

Procedure Using the First Derivative for Finding Extreme Values

- Find $f'(x)$.
- Solve the equation $f'(x) = 0$ to find those **critical points.**
- Include as **additional critical points** all points where $f'(x)$ does not exist (for whatever reason).
- Choose **test points** between adjacent critical points.
- Evaluate the sign ($+$ or $-$) of $f'(x)$ at each test point.
- A **relative maximum** occurs at a critical point c if
  - $f(c)$ exists and
  - $f'(x) > 0$ at the test point to the left of c (increasing) and
  - $f'(x) < 0$ at the test point to the right of c (decreasing).
- A **relative minimum** occurs at a critical point c if
  - $f(c)$ exists and
  - $f'(x) < 0$ at the test point to the left of c (decreasing) and
  - $f'(x) > 0$ at the test point to the right of c (increasing).

Example 3.1.2

Example 3.1.2

Find the extreme values of $f(x) = \dfrac{x^2}{x - 2}$.

Example 3.1.7

Example 3.1.7

The revenue, in millions of dollars, derived from the sale of a new kind of motorized skateboard t weeks after its introduction is given by

$$R(t) = \frac{63t - t^2}{t^2 + 63},$$

for $0 \leq t \leq 63$, million dollars.

Example 3.1.7

Example 3.1.7

The revenue, in millions of dollars, derived from the sale of a new kind of motorized skateboard t weeks after its introduction is given by

$$R(t) = \frac{63t - t^2}{t^2 + 63},$$

for $0 \leq t \leq 63$, million dollars.

(a) When does the maximum revenue occur?

Example 3.1.7

Example 3.1.7

The revenue, in millions of dollars, derived from the sale of a new kind of motorized skateboard t weeks after its introduction is given by

$$R(t) = \frac{63t - t^2}{t^2 + 63},$$

for $0 \leq t \leq 63$, million dollars.

(a) When does the maximum revenue occur?

(b) What is the maximum revenue?