Concavity

Lecture 27
Section 3.2

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Objectives

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- Understand what concavity is.
- Learn how to determine the concavity of a function.
- Understand what inflection points are and how to find them.

Concavity

Definition (Concave Upward)

The graph of a function $f(x)$ is **concave upward** on an interval if $f'(x)$ is increasing on that interval.

Definition (Concave Downward)

The graph of a function $f(x)$ is **concave downward** on an interval if $f'(x)$ is decreasing on that interval.

Example

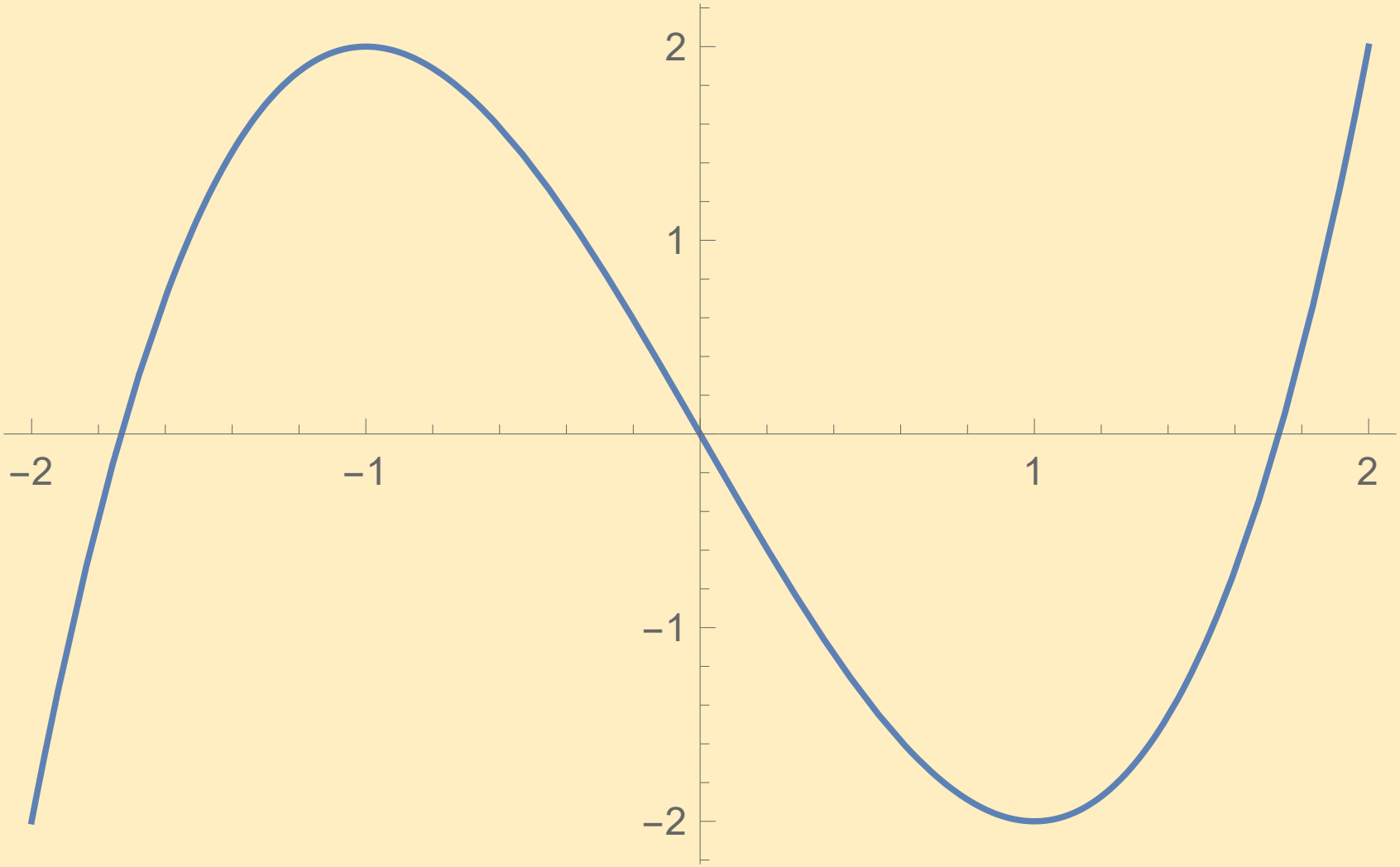
Example

Determine where the graph of the function

$$f(x) = x^3 - 3x$$

is concave upward and where it is concave downward.

The graph of $f(x) = x^3 - 3x$ on $-2 \leq x \leq 2$



Inflection Points

Definition (Inflection Point)

An **inflection point** of a graph is a point where the concavity of the graph changes either from upward to downward or from downward to upward.

Example 3.2.1

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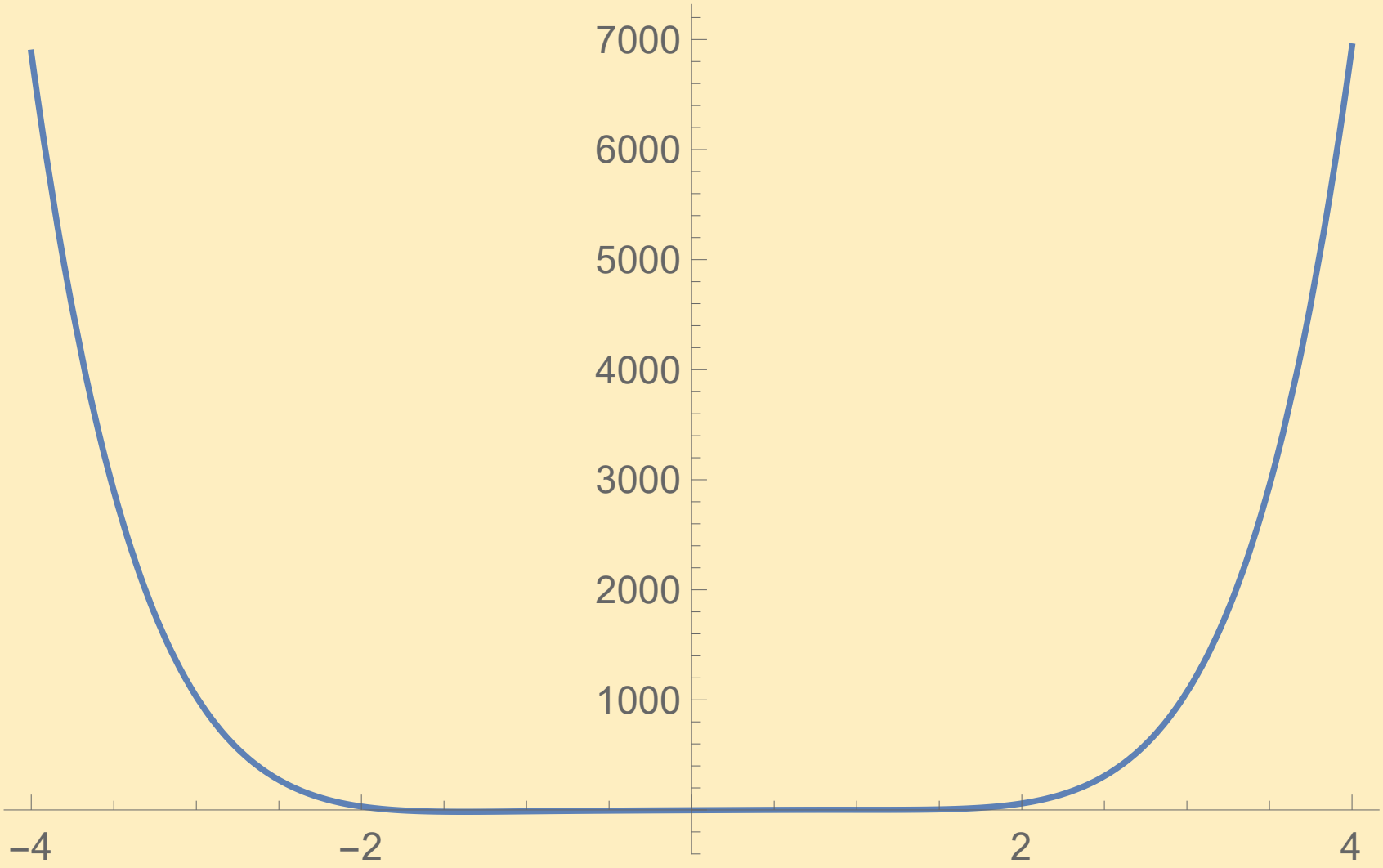
Determine where the graph of the function

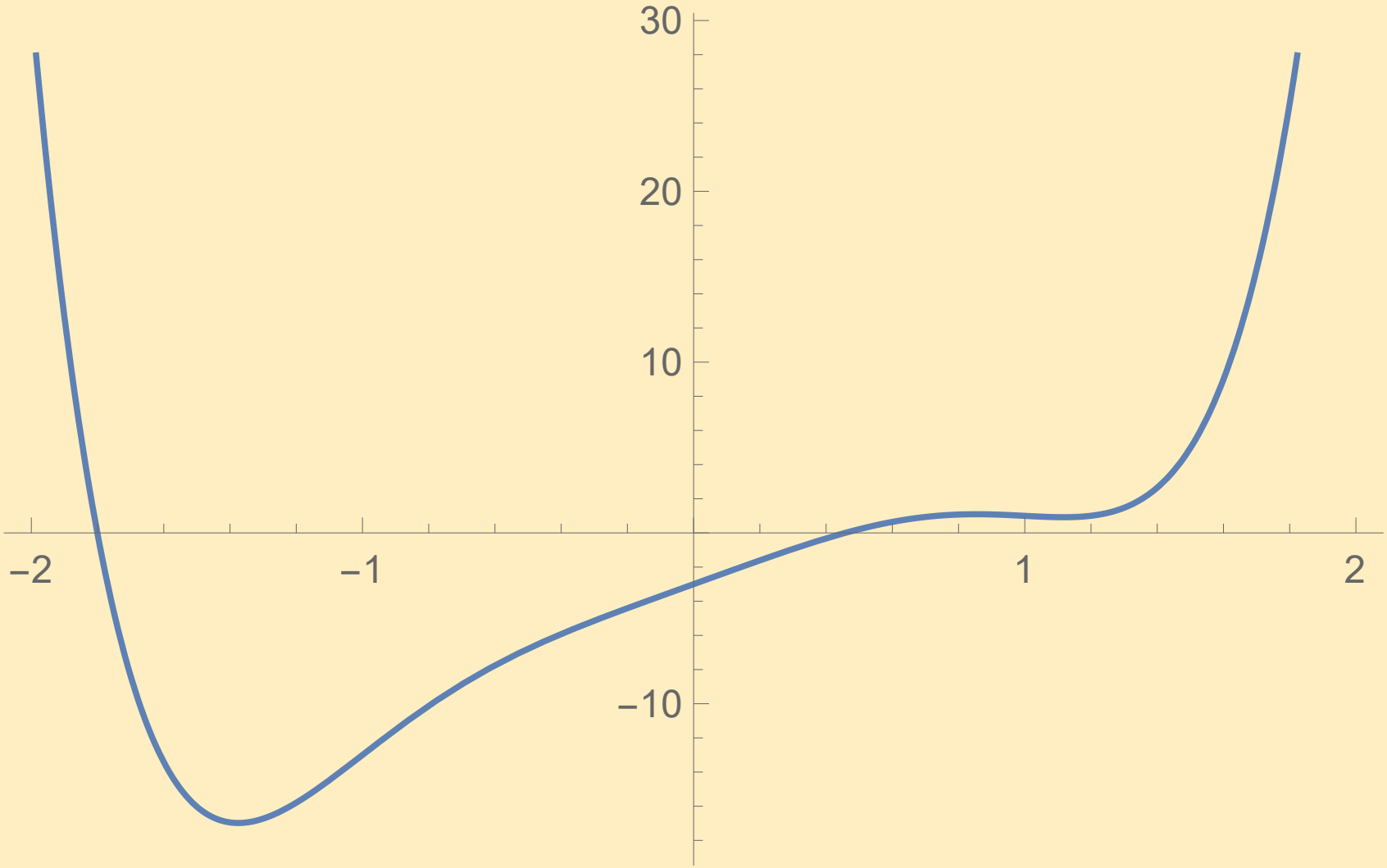
$$f(x) = 2x^6 - 5x^4 + 7x - 3$$

is concave upward and where it is concave downward.

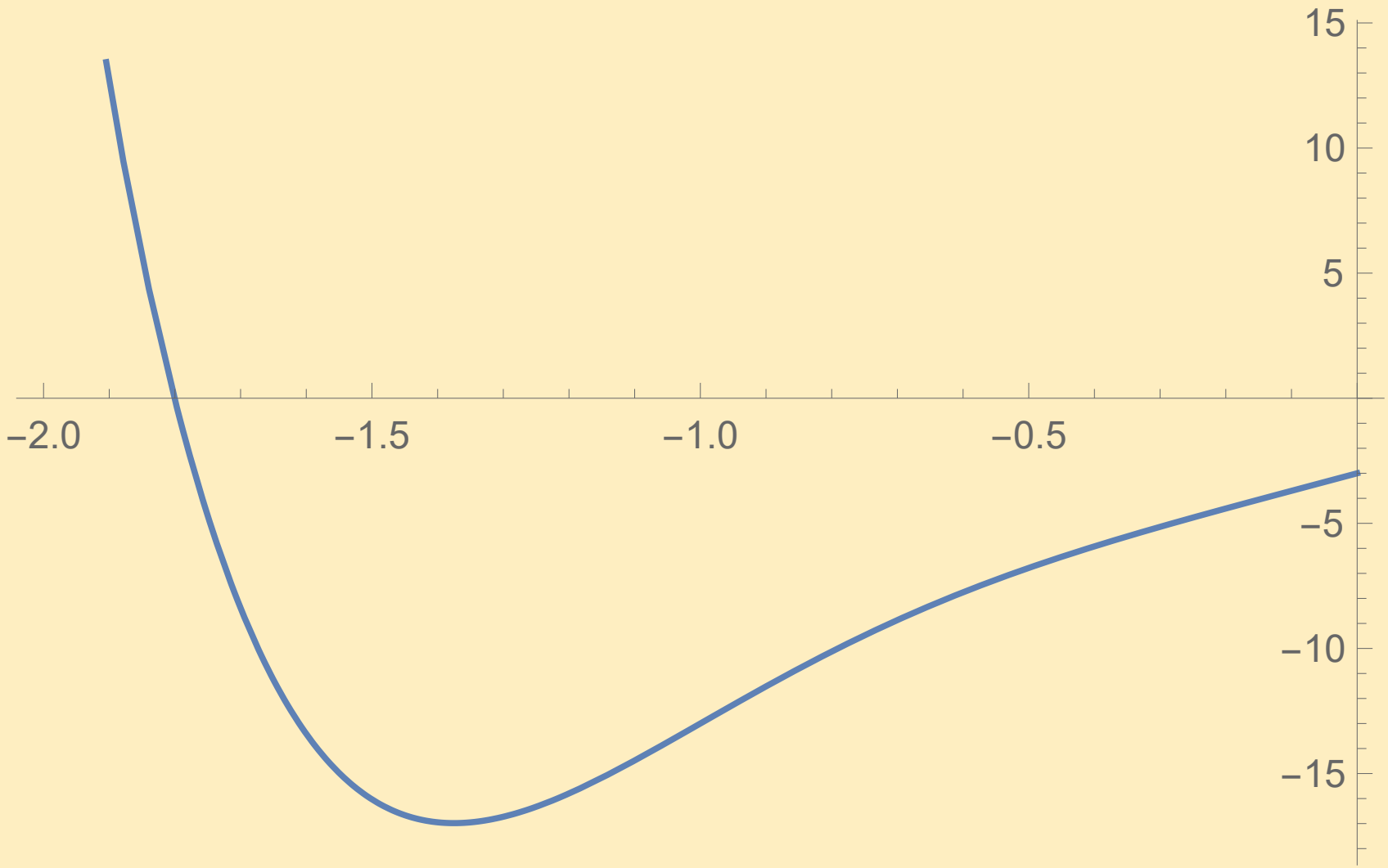
The graph of $f(x) = 2x^6 - 5x^4 + 7x - 3$ on $-4 \leq x \leq 4$



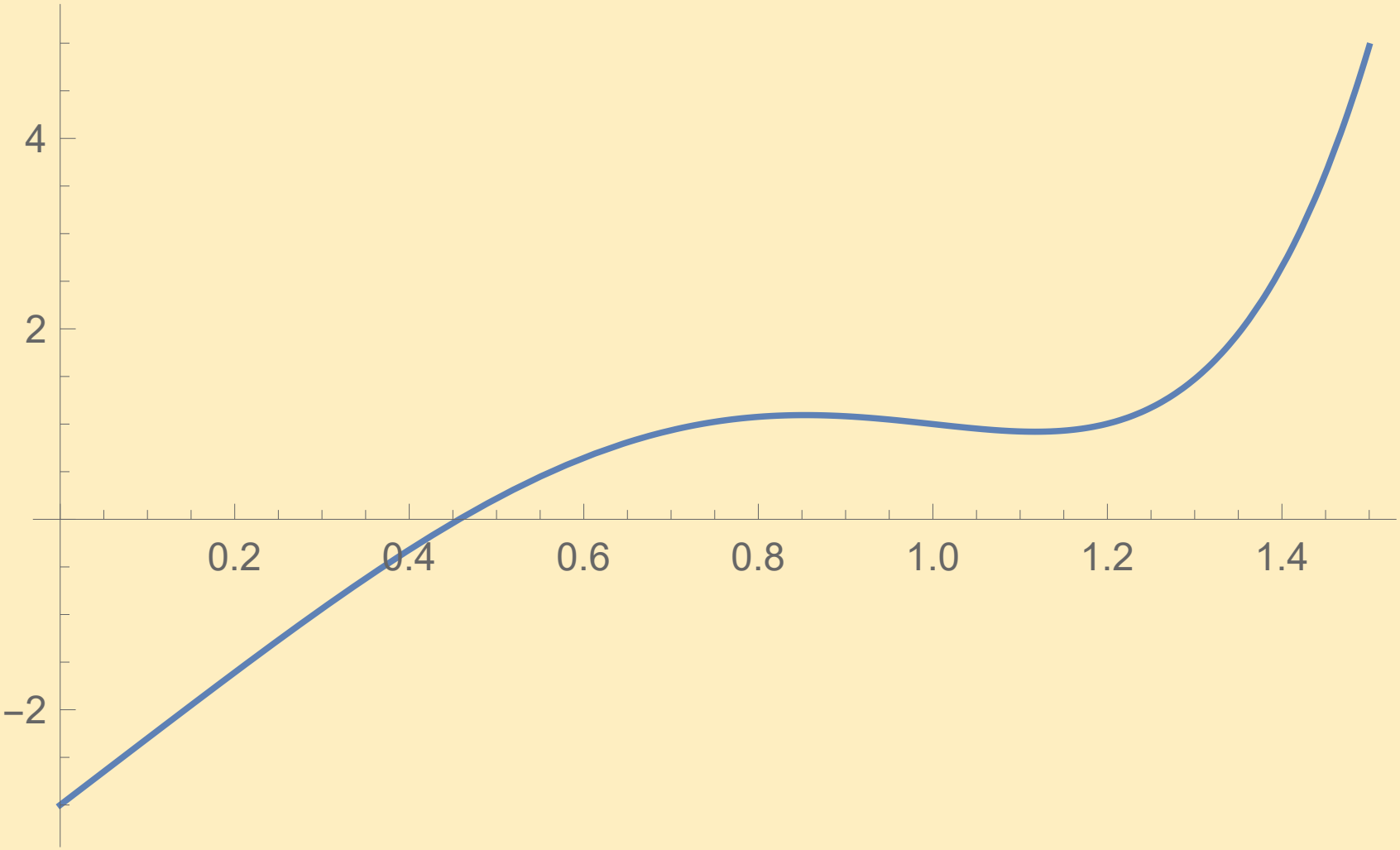
The graph of $f(x) = 2x^6 - 5x^4 + 7x - 3$ on $-2 \leq x \leq 2$



The graph of $f(x) = 2x^6 - 5x^4 + 7x - 3$ on $-2 \leq x \leq 0$



The graph of $f(x) = 2x^6 - 5x^4 + 7x - 3$ on $0 \leq x \leq 1.5$



Example

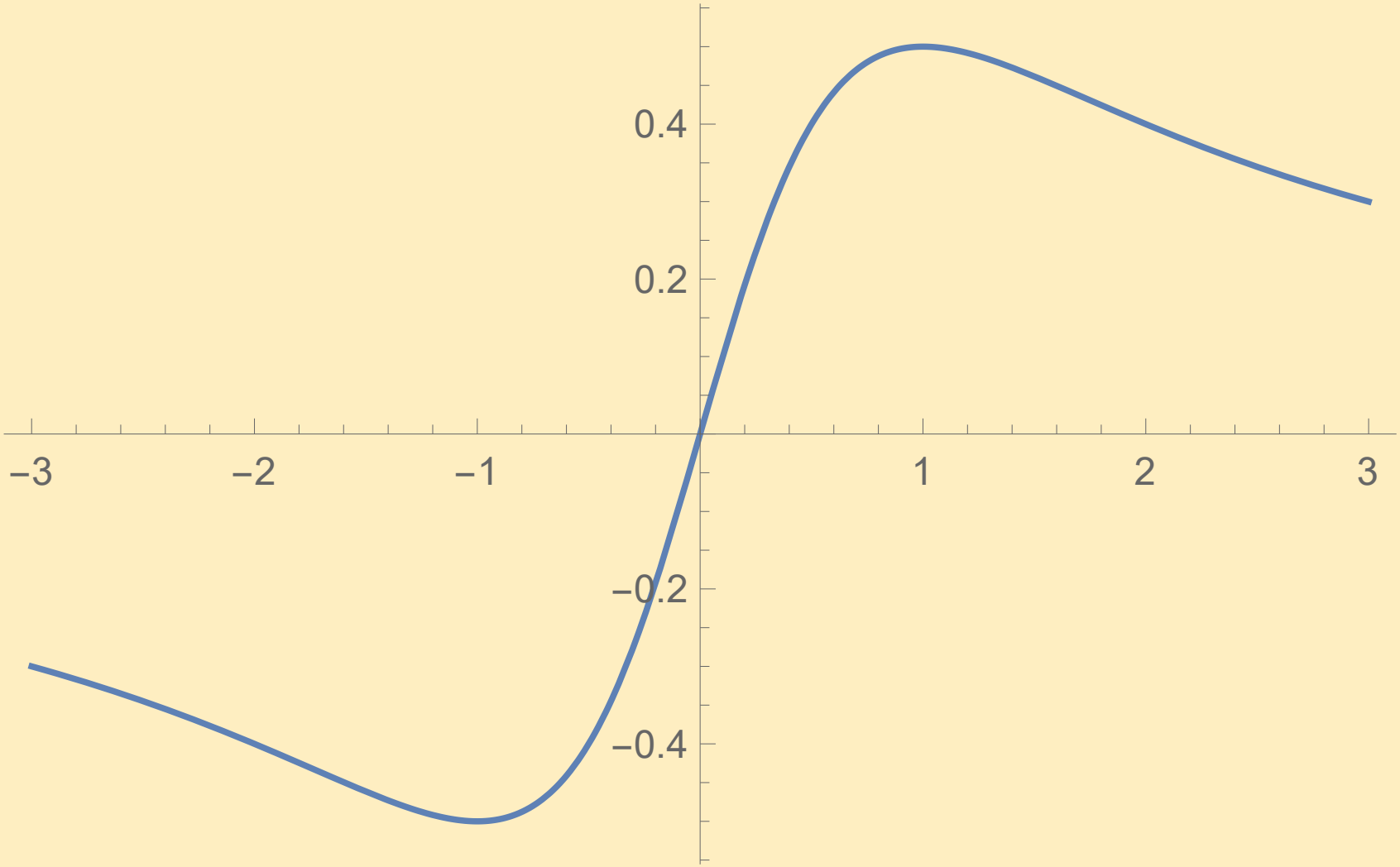
Example

Determine where the graph of the function

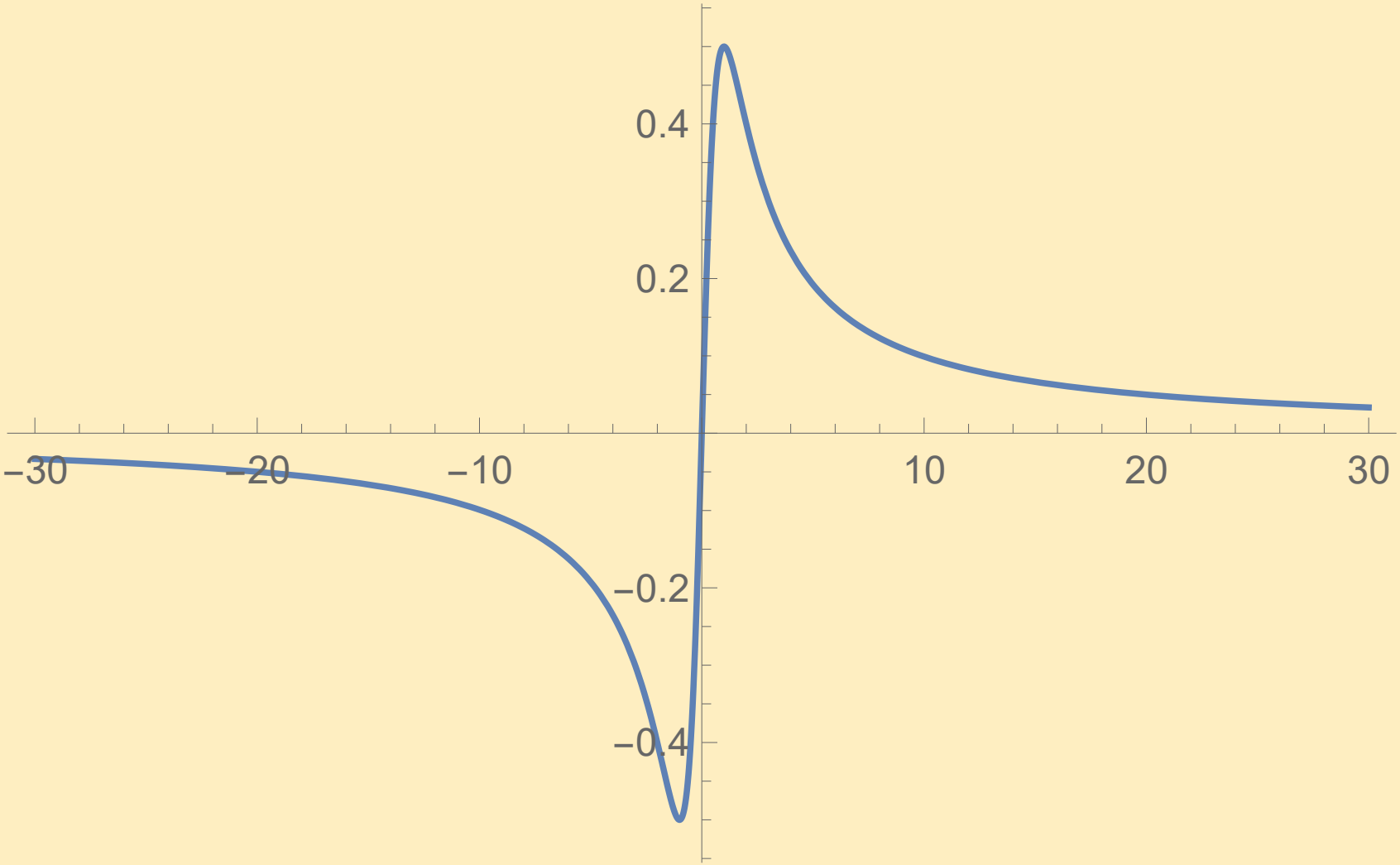
$$f(x) = \frac{x}{x^2 + 1}$$

is concave upward and where it is concave downward.

The graph of $f(x) = \dfrac{x}{x^2+1}$ on $-3 \leq x \leq 3$



The graph of $f(x) = \dfrac{x}{x^2+1}$ on $-30 \leq x \leq 30$



Example

Example

Determine where the graph of the function

$$f(x) = \frac{x}{x^2 - 1}$$

is concave upward and where it is concave downward.

The graph of $f(x) = \dfrac{x}{x^2 - 1}$ on $-3 \leq x \leq 3$