

Optimization

Lecture 30

Section 3.4

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Objectives

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- Use information derived from $f(x)$, $f'(x)$, and $f''(x)$ to
 - Maximize the revenue function, profit function, etc.
 - Minimize the cost function, average cost function, etc.
 - Maximize the rate of increase of the profit function.
 - Minimize the rate of increase of the cost function.

Example - Diminishing Returns of Production

Exercise 3.2.57

An efficiency study of the morning shift at a factory (7:00 am to 12:00 noon) indicates that an average worker who arrives on the job at 7:00 am will have produced Q units t hours later, where

$$Q(t) = -t^3 + \frac{9}{2}t^2 + 15t.$$

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- (a) When during the morning production does the worker's production read the point of diminishing returns?
- (b) When during the morning shift is the worker performing least efficiently?

Example - Cost Management

Exercise 3.3.51

A company uses a truck to deliver its products. To estimate costs, the manager models gas consumption by the function

$$G(x) = \frac{1}{2000} \left(\frac{800}{x} + 5x \right)$$

gal/mile, assuming that the truck is driven at a constant speed of x miles per hour for $x \geq 5$. The driver is paid \$18 per hour to drive the truck 400 miles, and gasoline costs \$4.25 per gallon. Highway regulations require $30 \leq x \leq 65$.

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(a) Find an expression for the total cost $C(x)$ of the trip.

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- Find an expression for the total cost $C(x)$ of the trip.
- Sketch the graph of $C(x)$ for the legal speed interval $30 \leq x \leq 65$.

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- Find an expression for the total cost $C(x)$ of the trip.
- Sketch the graph of $C(x)$ for the legal speed interval $30 \leq x \leq 65$.
- What legal speed will minimize the total cost of the trip? What is the total cost?

Example - Average Profit

Exercise 3.4.31

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- Show that average profit is maximized at the level of production \bar{q} found in part (b).
- On the same set of axes, graph the relevant portions of the average and marginal profit functions.