Optimization

Lecture 31
Section 3.4

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Objectives

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- Use information derived from $f(x)$, $f'(x)$, and $f''(x)$ to
  - Maximize the revenue function, profit function, etc.
  - Minimize the cost function, average cost function, etc.
  - Maximize the rate of increase of the profit function.
  - Minimize the rate of increase of the cost function.

Example - Diminishing Returns of Production

Exercise 3.2.57

An efficiency study of the morning shift at a factory (7:00 am to 12:00 noon) indicates that an average worker who arrives on the job at 7:00 am will have produced Q units t hours later, where

$$Q(t) = -t^3 + \frac{9}{2}t^2 + 15t.$$

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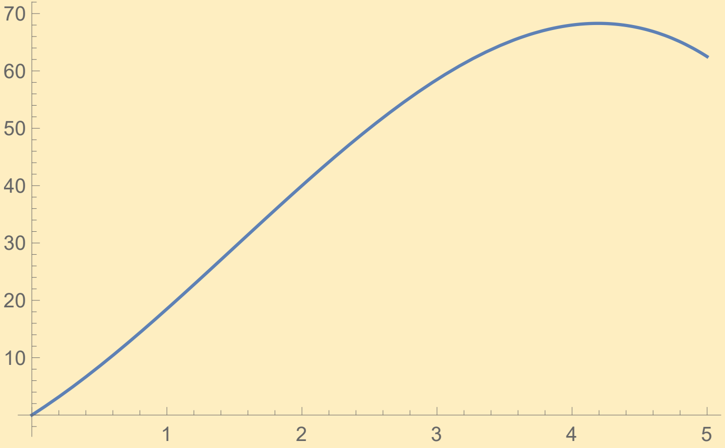
(a) When during the morning production does the worker's production read the point of diminishing returns?

(b) When during the morning shift is the worker performing least efficiently?

Exercise 3.2.57

The graph of $Q(t) = -t^3 + \frac{9}{2}t^2 + 15t$



Example - Cost Management

Exercise 3.3.51

A company uses a truck to deliver its products. To estimate costs, the manager models gas consumption by the function

$$G(x) = \frac{1}{2000}\left(\frac{800}{x} + 5x\right)$$

gal/mile, assuming that the truck is driven at a constant speed fo x miles per hour for $x \geq 5$. The driver is paid \$18 per hour to drive the truck 400 miles, and gasoline costs \$4.25 per gallon. Highway regulations require $30 \leq x \leq 65$.

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(a) Find an expression for the total cost $C(x)$ of the trip.

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(b) Sketch the graph of $C(x)$ for the legal speed interval $30 \leq x \leq 65$.

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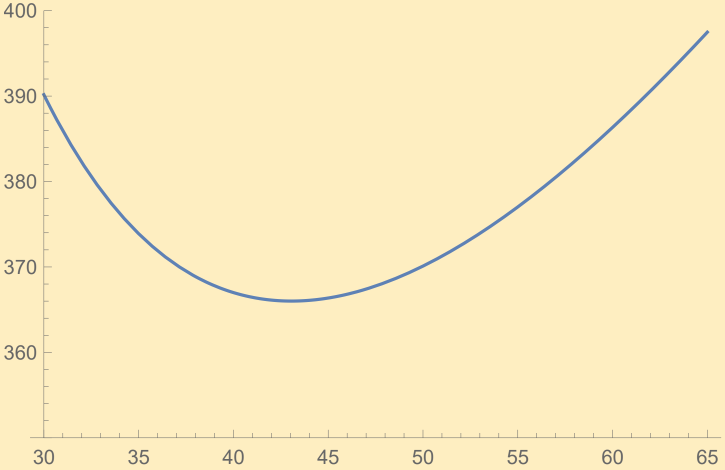
(a) Find an expression for the total cost $C(x)$ of the trip.

(b) Sketch the graph of $C(x)$ for the legal speed interval $30 \leq x \leq 65$.

(c) What legal speed will minimize the total cost of the trip? What is the total cost?

Exercise 3.3.51

The graph of $C(x) = \dfrac{7880}{x} + 4.25x$



Example - Average Profit

Exercise 3.4.31

A manufacturer estimates that when q units of a certain commodity are produced, the profit obtained is $P(q)$ thousand dollars, where

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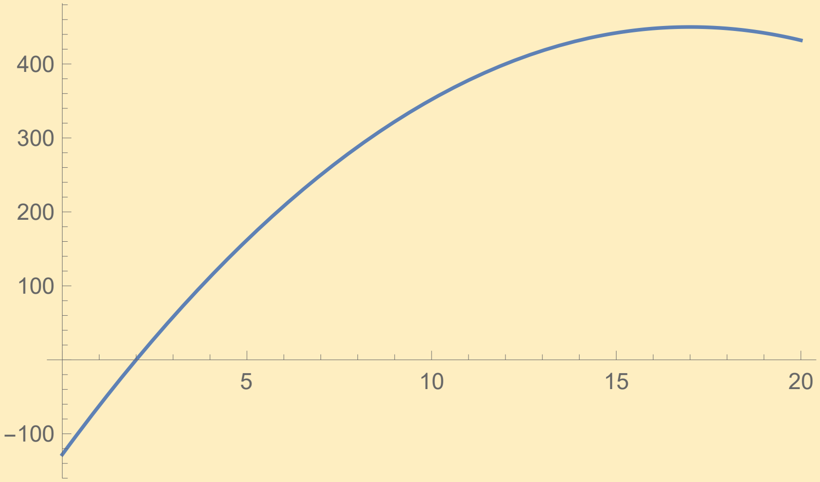
(c) Show that average profit is maximized at the level of production $\overline{q}$ found in part (b).

(d) On the same set of axes, graph the relevant portions of the average and marginal profit functions.

Exercise 3.3.51

The graph of the profit function $P(q) = -2q^2 + 68q - 128$



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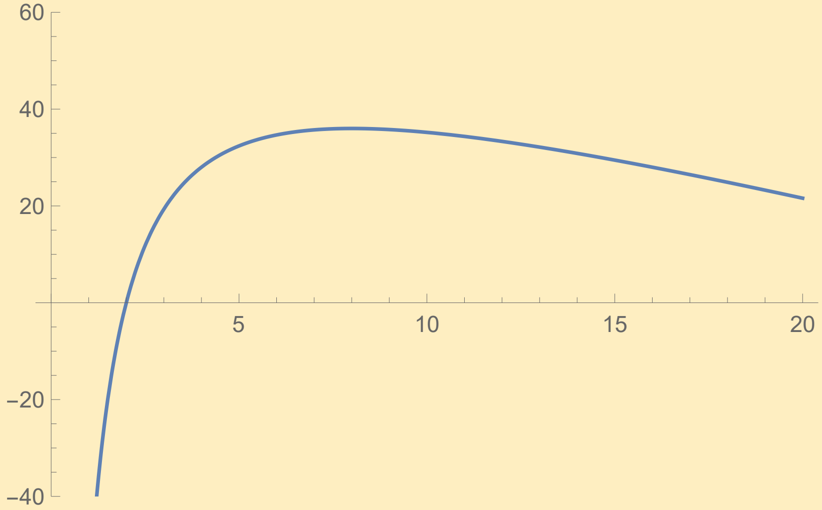
The graph of the average profit function



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The graphs of the average profit and marginal profit functions