

Elasticity

Lecture 32

Section 3.4

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Objectives

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- Understand why the average of a function attains its maximum (or minimum) when it equals its marginal value.
- Study the price elasticity of demand.
- Understand the effect of elasticity on revenue.

Average Cost vs. Marginal Cost

Theorem

Let $f(x)$ be a function and c a real number. If

$$\frac{f(c)}{c} = f'(c),$$

then $\frac{f(x)}{x}$ has a relative maximum at $x = c$.

Price Elasticity of Demand

Definition (Price Elasticity of Demand)

The **price elasticity of demand** is a measure of the amount of change (decrease) in demand is induced by a given change (increase) in price. Let p be the unit price and q the demand, in units sold, and let $q = D(p)$. Then the elasticity function $E(p)$ is defined as

$$E(p) = - \left(\frac{p}{q} \right) D'(p).$$

Percent Rate of Change

Definition (Percent Rate of Change)

The **percent rate of change** of a quantity $Q(x)$ is the rate of the change of $Q(x)$ relative to $Q(x)$ itself, expressed as a percent. That is,

$$\text{Percent rate of change of } Q(x) = 100 \cdot \frac{Q'(x)}{Q(x)}.$$

Example of Elasticity

Example 3.4.7 (modified)

The manager of a bookstore determines that when a certain new paperback novel is priced at p dollars per copy, the daily demand will be $q = 400 - p^2$ copies, where $0 \leq p \leq 20$.

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(a) Find the elasticity function of price on demand.

Levels of Elasticity

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At a given price p ,

- Demand is **elastic** if $E(p) > 1$.
- Demand is **inelastic** if $E(p) < 1$.
- Demand is **of unit elasticity** if $E(p) = 1$.

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- Find the elasticity function of price on demand.
- Determine where the demand is elastic, inelastic, and of unit elasticity with respect to price.

Effect of Elasticity on Revenue

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Let $R(p) = p \cdot D(p)$ be the revenue function. Then

$$\frac{dR}{dp} = q(1 - E(p)).$$

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- Find the elasticity function of price on demand.
- Determine where the demand is elastic, inelastic, and of unit elasticity with respect to price.
- Interpret the results of part (a) in terms of the behavior of total revenue as a function of price.