Compound Interest

Lecture 34
Section 4.1

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Reminder

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- Test #3 is this Friday, March 31.

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- Be there.

Objectives

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- Learn about the “natural” base.
- Apply exponential functions to compound interest.

The Natural Base e

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- Let $f(x) = b^x$ for some base $b > 0$.
- What is $f'(x)$?

The Natural Base e

- If we apply the definition, we get

$$f'(x) =$$

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$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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$$
\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{b^{x+h} - b^x}{h}
\end{aligned}
$$

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- For the right choice of b, that value will be 1. How do we know that?
- That choice is approximately $2.718281828\ldots$.
- We call that number e, the **natural base**.

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Then

$$B(t) = P\left(1 + \frac{r}{k}\right)^{kt}.$$

Continuous Compounding

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If the interest is **compounded continuously**, then the future value is

$$B(t) = Pe^{rt}.$$